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Abstract

The literature on North-South trade has explored conditions under which international trade might magnify income disparities between the advanced North and the backward South. Little attention has yet been placed on the effect of trade on countries that do not display substantial dissimilarities concerning aggregate capital endowments. We show that even when no single country is technologically more advanced than any other one and productivity changes are uniform and identical in all countries, international trade may still be a source of income divergence when nonhomothetic preferences and quality ladders are jointly taken into account. Income divergence will be experienced when comparative advantages induce patterns of specialisation that, although optimal for each country at some initial point in time, do not offer the same scope for improvements in terms of subsequent quality upgrading of final products.

Keywords: International Trade, Nonhomothetic Preferences, Quality Ladders
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1 Introduction

In the past two decades a number of articles on international trade have started to acknowledge the importance of nonhomothetic preferences for capturing some relevant features of North-South trade – e.g., Flam and Helpman (1987), Stokey (1991), and Matsuyama (2000). These papers have developed tractable models that yield patterns of specialisation where richer countries produce and export goods with high income demand elasticity. One of the main predictions of those models is that the impact of international trade on growth may be uneven across countries which are at different stages in the process of development. More precisely, trade would tend to be more beneficial to developed economies (the North), and it may even be detrimental to underdeveloped countries (the South). The key mechanism at work is the one originally proposed by Prebisch (1950) and Singer (1950): as the world income rises, world aggregate demand shifts towards the goods produced by the North, improving their terms of trade and, thereby, magnifying initial income disparities between those richer economies and the South.

The papers mentioned above thus restrict the attention to a world economy where some countries (the North) have somehow historically accumulated larger amounts of human and physical capital than others (the South), and show conditions under which trade magnifies initial income disparities resulting from those capital differences. However, the pattern of international specialisation and trade might also be the source of income differentials between countries that do not display any substantial dissimilarity regarding their initial levels of human and physical capital. In this paper, we look at economies that start off with similar capital endowments, and propose a theory of uneven growth induced by trade, based on nonhomothetic preferences, quality differentiation and productive specialisation driven by Ricardian comparative advantages.

Our theory rests on five fundamental elements. First, there exists a large number of consumption goods in the economy. Second, each specific type of consumption good is present in several levels of quality, with higher qualities being increasingly costly to produce. Third, some goods offer larger scope for quality upgrading than others, in the sense that it is less costly to increase their quality. Fourth, individuals care about the quality of the goods they consume and, moreover, their willingness to pay for higher quality of consumption increases with their income. Fifth, countries that are similar in terms of their average productivities specialise in the production of different goods according to their comparative advantage.

The first four elements above give room for nonhomothetic demand schedules, where the income demand elasticity of every good is tied to the specific quality in which that particular good is (optimally) traded in the market. The last element yields patterns of regional specialisation that, combined with nonhomothetic demand schedules, may lead to divergent dynamics among countries that are initially similar in terms of capital endowments. In such a framework, we show that international trade may induce income divergence across countries characterised by similar initial income levels and with no *absolute* advantage over one another. In particular, income divergence will be experienced when comparative advantages dictate patterns of specialisation that, although optimal for each specific country at a given stage of development, do not offer the same scope for technological improvements in terms of subsequent quality upgrading of final goods.

To convey some preliminary intuition of how nonhomothetic demand schedules arise as an equilibrium result of our model, it is worth discussing in further detail some of the specificities of the commodity space. In that respect, we follow the quality ladder structure featured in Grossman and Helpman (1991) – that is, in a continuum of horizontally differentiated goods, an infinite number of qualities for each good are available in the market. Unlike Grossman-Helpman, however, in our framework the optimal expenditure shares across goods do not remain constant as income changes. In particular, we postulate that the additional utility the individual derives from a marginal increase in the quality of the goods he consumes increases with the quantity of consumption, hence with the individual's income (in other words, the individual's *taste for quality* increases with income). As a result, as individuals become richer they optimally shift resources towards those goods whose quality can be set at relatively higher levels. The budget constraint, in turn, implies that the extent by which quality can be raised for any given type of good is related to its specific cost of quality upgrading. Thus, the distribution of quality upgrading across goods results from the interplay between the underlying technological structure and the response of the consumers' *taste for quality* to income variations.

In such a framework we show that, if the cost of quality upgrading differs across goods, then the shift towards higher-quality goods with rising income will (optimally) occur at different speeds across goods. More precisely, the lower the cost of quality upgrading for a specific good, the larger the quality upgrading for that good. This uneven climbing-up-the-quality-ladder will in turn lead to nonhomothetic demand schedules, where the fraction of income spent in different goods depends on the level of income itself.

When introduced into a general equilibrium model of international trade, the interplay between quality upgrading and comparative advantage may lead to income divergence through its effect on the terms of trade. To briefly characterise this mechanism, take some hypothetical country (call it country Z) that specialises in the production of good x, which exhibits high cost of quality upgrading. According to the mechanism proposed in this paper, quality upgrading for x is relatively slow as world income grows. Hence, the world expenditure share on x decreases over time, while it shifts towards goods whose quality can be upgraded faster. As a result, as the world income rises, Z experiences a decline in its terms of trade, because the types of goods it produces display low income demand elasticity.

An important and novel feature of our model is the fact that quality upgrading is a phenomenon that occurs *within* types of goods, and (possibly) heterogeneously *across* different types of goods. This, in turn, implies that our model generates two distinct (yet interrelated) types of nonhomothetic behaviour: first, nonhomotheticity *within* goods arises as richer consumers shift their expenditure towards higher-qualities of each specific good; second, nonhomotheticity *across* goods (possibly) arises as richer consumers shift their expenditure towards goods with larger scope for quality upgrading.[1]

Our open economy model predicts that richer economies specialise in the production of goods that exhibit larger scope for quality upgrading (in turn, implying that they specialise in the production of higher-quality goods). This prediction is in fact consistent with the evidence presented in Khandelwal (2009). This paper estimates the length of quality ladders for different industries, showing that import penetration from poorer economies in the US is lower in industries that exhibit longer quality ladders (hence larger scope for quality upgrading), while exports to the US originating from other developed economies tend to belong precisely to those industries and, in particular, to the upper spectrum of their respective (long) quality ladders.[2]

The model also predicts that citizens from richer countries consume higher qualities than those consumed by citizens from poorer countries. This prediction rationalises the findings by Verhoogen (2008) and Iacovone and Javorcik (2009), who show that Mexican manufacturing plants produce higher-quality goods to export to richer markets (mainly the US), and by Hallak

[1] Notice that we keep the word 'possibly' in parenthesis for a very specific reason: if all goods offered the same scope of quality upgrading, then our model would no longer feature the second source of nonhomotheticity (i.e., expenditure shares would remain constant across types of goods with rising income).

[2] Schott (2004) also shows that the quality dimension within varieties of goods (measured in his paper by unit values) is key to understand trade patterns in the world economy. In particular, he provides evidence that US import unit values correlate positively with the exporter's GDP per head. Moreover, this positive correlation tends to be more pronounced for goods that exhibit larger scope for quality upgrading (e.g., manufactured goods) compared to more homogeneous goods (e.g., natural resources goods). See also Hallak (2006) for related evidence showing that rich countries import relatively more from countries that produce higher-quality goods.

and Schott (2009) who, using cross-country data, show that the quality gap in production between rich and poor economies is smaller than their income gap, which suggests that poorer economies are producing high-quality goods to sell in richer markets.[3]

Most of the existing trade literature with nonhomothetic preferences has relied on specifications that take luxuries as an exogenous category, like hierarchical or "0/1" preferences.[4] An implication of this exogeneity of luxuries is the fact that they can only deliver hump-shaped Engel curves. By contrast, our utility specification is able to let different goods behave as luxuries at different levels of income, and hence rationalise the highly non-monotonic shape that observed Engel curves take, even after controlling for a number of factors such as consumers' age and households' composition (for a short review of the findings about empirical Engel curves, see Lewbel, 2006).

An exception to the above literature is a recent paper by Fajgelman, Grossman and Helpman (2009), who provide a model of international trade with nonhomothetic preferences and differentiated goods, which can be offered in several degrees of quality. Different from our paper, their production technology is the same for all types of goods. Hence, nonhomotheticity is unrelated to the heterogeneous scope for quality upgrading across goods, which is a crucial point in our model. Finally, our paper also relates to other contributions that study Ricardian trade models with quality ladders for a continuum of differentiated types of goods, such as Taylor (1993), Alcala (2009), and Benedetti Fasil and Borota (2009). All these contributions, however, use homothetic specifications of preferences.

The paper is organised as follows. Section 2 describes the setup of the model. Section 3 presents the partial equilibrium consumer's problem, illustrating the specificities of the nonhomotheticity of demand in our model. Section 4 computes the general equilibrium in the world economy, and analyses the effects of uniform aggregate productivity growth, population growth and income inequality within countries. Section 5 presents some illustrative empirical results consistent with the main model's predictions using cross-country trade data. Section 6 illustrates our theory with a particular historical example. Section 7 concludes. The appendices contain the omitted proofs and some additional algebraic derivations used in the main text.

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[3] Brooks (2006) provides evidence similar to Verhoogen's (2008) for Colombian manufacturing plants. The same conclusion as Hallak and Schott (2009) follows from Fieler (2007), who reports that unit prices (a proxy for quality) rise with the importer's income per capita, even for goods from the same exporter and category.

[4] Further details on hierarchical preferences can be found in Bertola, Foellmi and Zweimuller (2006, pp. 302-320). The "0/1" specification of preferences is due to Foellmi, Hepenstrick and Zweimuller (2008).

2 Structure of the Model

We consider a world composed by two countries: the *Home* country and the *Foreign* country. For brevity, hereafter we refer to the former as H and to the latter as F. These two economies share a common commodity space, defined along two distinct dimensions: *horizontal* and *vertical*. The first dimension (*horizontal*) designates the different types of goods (e.g., fruit products, TVs, etc.). Different goods are indexed by the letter v along the space $\mathbb{V} \subset \mathbb{R} : v \in [0, 1]$. The second dimension (*vertical*) refers to the intrinsic *quality* of the good of each particular type v (e.g., organic vs. non-organic fruit products, LCD TVs vs. cathode ray tube TVs, etc.). For each good $v \in \mathbb{V}$, commodities are *vertically* ordered by the quality-index q belonging to the set $\mathbb{Q} \subset \mathbb{R} : q \in [1, \infty)$, where a higher q denotes a higher quality. The commodity space is then given by the set $\mathbb{V} \times \mathbb{Q} = [0, 1] \times [1, \infty)$, and each commodity is identified by a pair $(v, q) \in \mathbb{V} \times \mathbb{Q}$.[5]

We assume that all commodities are tradable. Additionally, we assume there are no transport costs and no tariffs affecting international trade.

2.1 Technology

In both H and F competitive firms produce commodities based on linear production functions in which labour represents their only variable input. Whenever it proves needed, hereafter we adopt the following notation: unstarred symbols refer to H, starred ones to F. We let unit labour requirements vary both across goods and across qualities of each good. Also, we let unit labour requirements differ across countries. In particular, in H the unit labour requirement for commodity $(v, q) \in \mathbb{V} \times \mathbb{Q}$ is given by $c_{vq} = a(v) \, q^{\eta(v)}/\kappa$, while in F is given by $c^*_{vq} = a^*(v) \, q^{\eta(v)}/\kappa$.

The parameter $\kappa > 0$ above denotes a *world aggregate-productivity* parameter, which can be interpreted as the *global technology frontier*. The functions $a(v)$ and $a^*(v)$ represent *good-specific* technological parameters, for H and F respectively, and we assume they may differ between those two economies. Finally, the function $\eta(v)$ summarises the *cost elasticity of quality upgrading* for

[5] In our setup, different goods should be then understood as groups of commodities that aim at satisfying different *needs*. On the other hand, different qualities of a particular good refer to the *extent* (or *degree*) in which the *need* is actually satisfied by the commodity. In that regard, food satisfies a different need when compared to TVs (physiological nutrition vs. visual entertainment), but LCD TVs satisfy the need for visual entertainment (objectively!) better than cathode ray tube TVs.

each good v, which is assumed to be the same for both H and F. Henceforth, we suppose that $a(v) : [0,1] \rightarrow \mathbb{R}_{++}$, where $a'(\cdot) \geq 0$; analogously, $a^*(v) : [0,1] \rightarrow \mathbb{R}_{++}$, where $a^{*\prime}(\cdot) \geq 0$. We also assume that $\eta(v) : [0,1] \rightarrow \mathbb{R}_{++}$, where $\eta'(\cdot) > 0$ and $\eta(0) > 1$.[6]

In our world economy, each country will naturally specialise in those commodities they can produce more cheaply. As a result, the *international* price of each commodity will be given by $p_{vq} = \min\left\{c_{vq}w, c^*_{vq}w^*\right\}$, where w (w^*) denotes the wage in H (F), measured in a common *numeraire.* Given the unit labour requirements in the two countries specified above, we can express the international price of each commodity $(v,q) \in \mathbb{V} \times \mathbb{Q}$ as follows:

$$p_{vq} = \alpha(v)\, q^{\eta(v)}/\kappa, \tag{1}$$

where $\alpha(v) \equiv \min\{a(v)\, w, a^*(v)\, w^*\}$.

2.2 Preferences and Budget Constraint

Both H and F are inhabited by a continuum of individuals with identical preferences defined over the commodity space $\mathbb{V} \times \mathbb{Q}$.

We assume that individuals consume only one quality, denoted by q_v, of each type of good v. Let $x_v \in \mathbb{R}_+$ denote the consumed quantity of commodity q_v (i.e., the consumed quantity of good v in quality q) by a representative individual from H. This individual's preferences are summarised by the following utility function:

$$\begin{gathered} U = \int_{\mathbb{V}} \ln C_v \, dv \\ \text{with } \; C_v = \begin{cases} x_v & \text{if } \; x_v < 1 \\ (x_v)^{q_v} & \text{if } \; x_v \geq 1 \end{cases} \end{gathered} \tag{2}$$

where C_v represents a quality-adjusted consumption index.[7]

[6] From the labour requirements functions it is apparent that qualitative upgrade is costly, which seems a natural assumption to make. Additionally, from our assumptions it follows that $\eta(v) > 1$ for all $v \in \mathbb{V}$, which implies that the marginal cost of improving quality is, for each good, increasing along the quality space. In that sense, this assumption also seems quite natural, as it reflects the fact that subsequent quality improvements become increasingly costly. Finally, note that $\eta'(\cdot) > 0$, coupled with $a'(\cdot) \geq 0$, implies that goods are sorted along the space $\mathbb{V}$ by their cost of quality upgrading.

[7] The assumption of a single consumed quality for each good is posed to ease our exposition, and it corresponds to the solution that arises when assuming an infinite degree of substitution between qualities of the same goods. More precisely, the single consumed quality would still arise if we were to consider the following utility function: $U = \int_{\mathbb{V}} \ln \left[\int_{\mathbb{Q}} \max\{x_{vq}, (x_{vq})^q\}\, dq\right] dv$, where x_{vq} denotes the consumed quantity of commodity $(v,q) \in \mathbb{V} \times \mathbb{Q}$.

The utility function captures the notion that quality is a desirable feature, and that quality turns increasingly desirable as physical consumption rises. Notice that quality magnifies the utility derived from (physical) consumption only when $x_v > 1$. This last property of (2) intends to capture the idea that individuals first seek to satisfy their basic consumption needs, and just after these basic needs are met, do they start paying attention to the quality dimension of the goods they consume.

Some additional properties about the utility function specified in (2) are worth noting. First, for each good v, marginal utility is unbounded above as consumption approaches zero, implying that all goods will be actively consumed in an optimum. Second, considering the hypothetical consumed quantities, $x_{v\overline{q}}$ and $x_{v\underline{q}}$, of two different levels of the quality-index, $\underline{q} < \overline{q}$, for the same good v, the marginal rate of substitution of $x_{v\overline{q}}$ for $x_{v\underline{q}}$ is non-decreasing along a *proportional expansion path* of $x_{v\overline{q}}$ and $x_{v\underline{q}}$.[8] This last property of (2) allows demand functions to display nonhomothetic behaviour, where the rich spend a larger fraction of their income in higher-qualities than the poor.

Each individual is endowed with one unit of *effective* labour, which is supplied inelastically. Labour is immobile across countries. As a result, each individual in H supplies his entire labour endowment to domestic firms in return of a wage $w \in \mathbb{R}_{++}$. This wage represents the only source of income for the individual. Therefore, his budget constraint reads as follows:

$$\int_{\mathbb{V}} p_v \, x_v \, dv \leq w \tag{3}$$

where $p_v \in \mathbb{R}_{++}$ denotes the (international) price of each unit of good q_v.

We define $\beta_v \equiv p_v x_v / w$ as the *demand intensity* of good $v \in \mathbb{V}$.[9] In the optimum, given the specification in (2), the budget constraint (3) will naturally bind. It is thus straightforward to notice that demand intensities will sum up to one across goods (i.e., $\int_{\mathbb{V}} \beta_v dv = 1$).

All individuals in the world face the same prices for the reproducible commodities. As a result, the analogous expressions in (2) and (3) corresponding to F read, respectively, as follows:

[8] To see this, note the $MRS(x_{v\overline{q}}, x_{v\underline{q}})$ is defined by $(\partial U/\partial x_{v\overline{q}}) / (\partial U/\partial x_{v\underline{q}})$ and, along a proportional expansion path, $x_{v\overline{q}} = k \, x_{v\underline{q}}$, with $k > 0$. Then, from (2), for $x_{v\underline{q}}, x_{v\overline{q}} > 1$:

$$MRS(k \, x_{v\underline{q}}, x_{v\underline{q}}) = \left(\overline{q}/\underline{q}\right) k^{\overline{q}-1} \left(x_{v\underline{q}}\right)^{\overline{q}-\underline{q}},$$

from where it is clear that, along the *ray* $x_{v\overline{q}} = k \, x_{v\underline{q}}$, $MRS(x_{v\overline{q}}, x_{v\underline{q}})$ is increasing in $x_{v\underline{q}}$.

[9] Demand intensities are the continuous counterpart of the discrete-case expenditure shares. Their relationship is analogous to that between densities and discrete probabilities. We borrow this nomenclature from Horvath (2000).

$U^* = \int_{\mathbb{V}} \max\left\{x_v, (x_v)^{q_v^*}\right\} dv$ and $\int_{\mathbb{V}} p_v x_v^* dv \leq w^*$. (Bear in mind that, since labour is immobile, w and w^* need not be equal.)

3 The Individual's Optimal Consumption Choice

In this section we present the optimal consumption choice of a representative individual from H, given the set of prices in the world economy. The results so obtained can be easily extended to an individual from F, which is done in Appendix B.

Before stating the consumer's optimisation problem, it proves convenient to state the following preliminary result:

$$q_v > 1 \;\Rightarrow\; x_v > 1. \tag{4}$$

This result follows immediately from noting that, for all $v \in \mathbb{V}$, utility derived from consuming $x_v \in (0, 1]$ is independent of the consumed quality q_v, while according to (1) the price of commodity q_v is strictly increasing along the quality space. Given (4), we may then restate the quality-adjusted consumption index in (2) simply as: $C_v = (x_v)^{q_v}$.

Bearing in mind result (4) and the fact that $x_v = w\beta_v/p_v$, the individual's optimisation problem can be thus stated in terms of two sets of control variables, namely $\{q_v, \beta_v\}_{v\in\mathbb{V}}$:

$$\begin{aligned}
\max_{\{q_v,\beta_v\}_{v\in\mathbb{V}}} \quad & U = \int_{\mathbb{V}} q_v \ln\left(\frac{w\beta_v}{p_v}\right) dv, \\
\textit{subject to:} \quad & \int_{\mathbb{V}} \beta_v dv = 1, \\
& q_v \geq 1,\ \forall v \in \mathbb{V}, \\
& p_v = \alpha(v)\,(q_v)^{\eta(v)}/\kappa,\ \forall v \in \mathbb{V}.
\end{aligned} \tag{5}$$

The first-order conditions corresponding to (5) are stated in the Appendix A. From those first-order conditions we may obtain the following expression for each β_v in the optimum:

$$\beta_v = q_v/Q,\ \forall v \in \mathbb{V}, \tag{6}$$

where $Q \equiv \int_{\mathbb{V}} q_z dz$ can be regarded as an aggregate index measuring the optimal consumption bundle's *average quality.* Notice that, according to (6), the fraction of income spent on good v is determined by its optimal quality relative to the average quality of consumption. In that regard, if all goods were optimally consumed at identical quality degrees (i.e., if $q_v = Q$, $\forall v \in \mathbb{V}$), then $\beta_v = 1$ would hold for all $v \in \mathbb{V}$, and our model would behave exactly as the one by Dornbusch, Fischer and Samuelson (1977).

3.1 Distribution of Qualities and Demand Intensities across Goods

Given the technology in the world economy, summarised by κ, $\alpha(\cdot)$ and $\eta(\cdot)$, it is possible to characterise the distribution of the optimal qualities across goods according to their position within the set $\mathbb{V}$. Lemma 1 provides the first result in that direction.

Lemma 1

Consider two goods $\underline{v}, \overline{v} \in \mathbb{V}$, such that $\underline{v} < \overline{v}$. Then: $q_{\underline{v}} \geq q_{\overline{v}}$; with strict inequality iff $q_{\underline{v}} > 1$.

Proof. See Appendix C. ■

Lemma 1 implies that the consumed quality q_v is non-increasing in the good-index v. The underlying intuition for Lemma 1 is straightforward: those goods which can be more cheaply upgraded tend to be optimally consumed in higher quality degrees.

The monotonicity of q_v implied by Lemma 1 allows us to split the goods space in two disjoint subsets. The first subset containing goods that are bound to be consumed at the baseline quality (i.e., $q_v = 1$) – these are the higher-indexed goods. The second one comprising the goods for which the constraint $q_v \geq 1$ in (5) does not bind in the optimum – these are the lower-indexed goods. Henceforth, we denote the second subset by $\mathbb{L} \subseteq \mathbb{V}$.

Lastly, regarding the distribution of the demand intensities, from the condition in (6) we can observe that, in the optimum, demand intensities are set proportional to the optimal qualities. As a result, the distribution of β_v across goods will qualitatively mirror that of q_v.

3.2 Effects of Aggregate Productivity Growth on Demand

In this section we study the effects of letting the parameter κ vary, while holding unchanged the functions $a(\cdot)$, $a^*(\cdot)$ and $\eta(\cdot)$, along with w and w^*. The consequence of this is letting the consumer's real income increase, without altering any of the relative prices of commodities in the space $\mathbb{V} \times \mathbb{Q}$.

For sufficiently low levels of aggregate productivity, the subset of goods consumed at the *baseline* quality initially comprises the entire set $\mathbb{V}$; formally, $\mathbb{L} = \emptyset$ holds when κ is below the threshold $\underline{\kappa} \equiv a(0)\exp(\eta(0))$. As world aggregate productivity rises beyond the threshold $\underline{\kappa}$, the subset $\mathbb{L}$ starts expanding, and eventually $\mathbb{L} = \mathbb{V}$ holds when κ is sufficiently large.[10]

The next lemma complements Lemma 1 and describes in further detail how optimal qualities evolve as the parameter κ changes.

[10] For a formal proof of these results, see Lemma 3 in Appendix D.

Lemma 2

Let $\mathbb{L} = \{v \in \mathbb{V} : \lambda_v = 0\}$*, where* λ_v *is the Lagrange multiplier associated to the constraint* $q_v \geq 1$*. Consider two goods* $\underline{v}, \overline{v} \in \mathbb{V}$*, such that* $\underline{v} < \overline{v}$*. Then:*

i) $\forall \kappa \in (0, \underline{\kappa}) \;\; \Rightarrow \;\; \partial q_{\underline{v}} / \partial \kappa = \partial q_{\overline{v}} / \partial \kappa = 0$;

ii) $\forall \kappa \geq \underline{\kappa} \;\; \Rightarrow \;\; \partial q_{\underline{v}} / \partial \kappa \geq \partial q_{\overline{v}} / \partial \kappa$; *with strict inequality if* $\underline{v} \in \mathbb{L}$*.*

Proof. See Appendix C. ■

Lemma 2 shows that, whenever $\mathbb{L}$ is non-empty (i.e., case *ii* in the lemma), for all goods belonging to $\mathbb{L}$ the consumed quality increases when world aggregate productivity rises. Furthermore, this effect is stronger for those goods whose quality can be more cheaply upgraded – i.e., those goods carrying a lower $\eta(v)$. On the other hand, we can observe that the optimal quality of goods that do not belong to $\mathbb{L}$ does not respond to (infinitesimal) changes in κ.

We can accordingly identify two distinct regimes depending on the level of κ that prevails. First, we refer to an economy with $\kappa \leq \underline{\kappa}$ as a *subsistence economy.* In a subsistence economy, all goods are consumed at the baseline quality. Second, we refer to an economy with $\kappa > \underline{\kappa}$ as a *modern economy.* In a modern economy some goods (and possibly all of them) are consumed strictly above the baseline quality.

In what follows we proceed to further characterise these two regimes.

Subsistence Economy: $\kappa \leq \underline{\kappa}$

In this regime, $q_v = 1$ holds for all $v \in \mathbb{V}$. This in turn means that $Q = 1$ and $\beta_v = 1$ must hold for all $v \in \mathbb{V}$ as well. Thus, in a subsistence economy demand intensities remain constant and equal to one for all goods as κ increases.[11] In that regard, a subsistence economy displays analogous behaviour to the economy discussed in Dornbusch *et al* (1977), where demand schedules are homothetic across types of goods.

Modern Economy: $\kappa > \underline{\kappa}$

This regime is characterised by $q_v > 1$ for all $v \in [0, \tilde{v}(\kappa))$, where $\tilde{v}(\kappa)$ denotes the threshold $v \in \mathbb{V}$ such that $q_v > 1$ for all $v < \tilde{v}(\kappa)$. Hence, the average quality can be written as $Q = 1 - \tilde{v}(\kappa) + \int_0^{\tilde{v}(\kappa)} q_z \, dz$, from where it follows that $\partial Q / \partial \kappa = \int_0^{\tilde{v}(\kappa)} (\partial q_z / \partial \kappa) \, dz > 0$. Since $\partial q_v / \partial \kappa = 0$ for all $v \notin \mathbb{L}$, then because of (6), $\partial \beta_v / \partial \kappa < 0$ must hold for all $v \notin \mathbb{L}$. As a

[11] It must be noted that this result applies only if $\kappa \leq \underline{\kappa}$ holds *after* performing the comparative statics exercise.

result, given that $\int_{\mathbb{V}} \beta_v \, dv = 1$, it must thus be the case that the demand intensities of some (and possibly all) $v \in \mathbb{L}$ will increase as κ rises. Henceforth, let $\mathbb{J} \subset \mathbb{V}$ denote the subset of $\mathbb{V}$ comprising all those goods for which $\partial\beta_v/\partial\kappa > 0$.

In a subsistence economy, $\mathbb{J} = \emptyset$, while in a modern economy, $\mathbb{J} \neq \emptyset$. In other words, in a modern economy the homotheticity of demand intensities across goods no longer holds, as a subset of goods whose income demand elasticity is larger than one shows up. Notice, too, that $\mathbb{J} \subseteq \mathbb{L}$, since $\partial q_v/\partial\kappa > 0$ is a necessary condition for $\partial\beta_v/\partial\kappa > 0$ to hold.

The next proposition further characterises the behaviour of the demand intensities, β_v, as κ rises.

Proposition 1

Let $\mathbb{J} = \{v \in \mathbb{V} : \partial\beta_v/\partial\kappa > 0\}$. *Consider two goods* $\underline{v}, \overline{v} \in \mathbb{V}$, *such that* $\underline{v} < \overline{v}$. *Then:*

i) $\forall\kappa \in (0, \underline{\kappa}) \;\; \Rightarrow \;\; \partial\beta_{\underline{v}}/\partial\kappa = \partial\beta_{\overline{v}}/\partial\kappa = 0$;

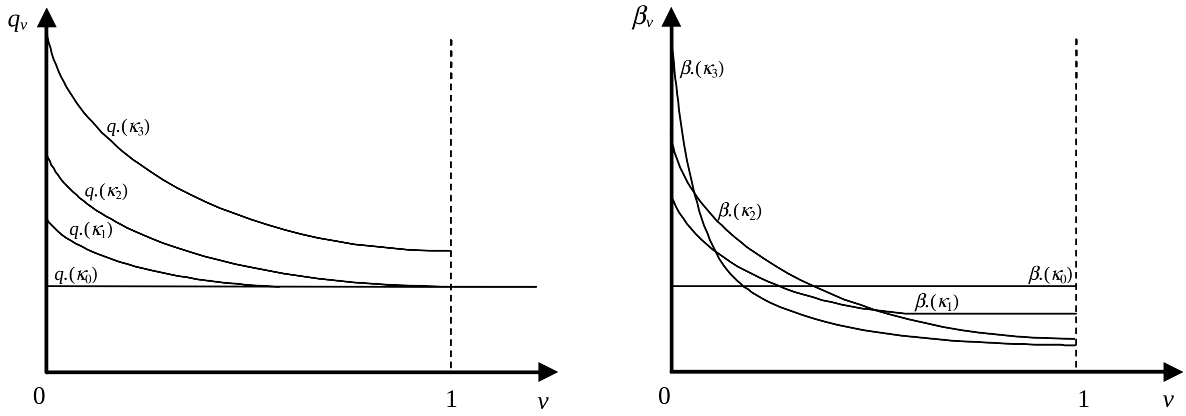
ii) $\forall\kappa \geq \underline{\kappa} \;\; \Rightarrow \;\; \partial\beta_{\underline{v}}/\partial\kappa \geq \partial\beta_{\overline{v}}/\partial\kappa$; *with strict inequality if* $\underline{v} \in \mathbb{J}$

Proof. See Appendix C. ■

To interpret our previous results more clearly, notice that $\mathbb{J}$ may be understood as the set of *luxury goods*, where by luxury goods we refer to those goods whose income demand elasticity is larger than 1. Since the set $\mathbb{J}$ always comprises lower-indexed goods, the luxury goods are exactly those goods whose quality q_v is relatively high compared to the average quality Q. In that regard, in our model it is the (relative) *quality* that determines whether or not a particular goods is *luxurious*.

Figure 1 illustrates this feature graphically. The distributions of qualities and demand intensities across goods are drawn for four different levels of world aggregate-productivity ($\kappa_0 \leq \underline{\kappa} < \kappa_1 < \kappa_2 < \kappa_3$). When individuals are still poor (i.e., for a level of productivity $\kappa_0 \leq \underline{\kappa}$), satisfying all basic needs constitutes their main goal, leading them to keep the quality of all goods at the baseline and setting accordingly equal demand intensities for all goods. As individuals become richer, some goods —for a level of productivity $\kappa_1 \in (\kappa_0, \kappa_2)$— and eventually all goods —for a level of productivity $\kappa_3 \geq \kappa_2$— are consumed in higher qualities. As a result, for those three levels of κ, a subset of goods with $\beta_v > 1$ appears in the lower spectrum of the (unit) goods set. Additionally, the goods whose quality is relatively higher attract increasingly larger income shares, as given the preference specification in (2) individuals tend to *value* high-quality commodities relatively more as they become wealthier. This last point is formalised in the following corollary.

Figure 1: Distribution of qualities and demand intensities across goods



Corollary 1

Let $\vartheta(v) \equiv \int_0^v \beta_z \, dz$. *Then:*

(i) $\forall \kappa \in (0, \underline{\kappa}) \;\Rightarrow\; \partial\vartheta(v)/\partial\kappa = 0$, $\forall v \in \mathbb{V}$;

(ii) $\forall \kappa \geq \underline{\kappa} \;\Rightarrow\; \partial\vartheta(v)/\partial\kappa \geq 0$, $\forall v \in \mathbb{V}$; *with strict inequality if* $v < 1$.

Proof. See Appendix C. ■

Corollary 1 synthesizes the eventual nonhomothetic behaviour of the demand schedules implied by our model. More precisely, whenever $\kappa < \underline{\kappa}$, demand schedules are homothetic across goods. However, when κ lies above the threshold $\underline{\kappa}$, income starts being spent in growing proportion on lower-indexed goods.

4 General Equilibrium in the World Economy

In Section 3, we have studied the optimal consumption choice of an individual from H, taking the wages in H and in F, w and w^*, as exogenously given. (In Appendix B, we do the same for the case of an individual from F.) These wages in turn determine the prices of all reproducible commodities in the world economy through equation (1). Our former analysis has therefore yielded only partial equilibrium results.

The present section computes the general equilibrium in this world economy. This requires endogenising wages and, thereby, the prices of all reproducible commodities. Given that in a general equilibrium only relative prices are determined, we henceforth take the wage in F as the *numeraire*, by setting $w^* = 1$.

So far we have not put any structure in terms of comparative advantage. The next assumption dictates the pattern of comparative advantage across countries.

Assumption 1 *Let* $A(v) \equiv a^*(v)/a(v)$*. We suppose: (i)* $A'(v) < 0$*, and (ii)* $\exists\, v_0 \in (0,1) : A(v_0) = 1$.

Assumption 1 represents the *only* source of heterogeneity across countries in our model. In particular, this last assumption implies that H enjoys a comparative advantage in the production of lower-indexed commodities, while F has a comparative advantage in the production of upper-indexed commodities.

Note that given the cost functions c_{vq} and c^*_{vq} specified in section 2.1, because $\eta(v)$ is the same for H and F, the nature of comparative advantage does not change as we move up in the quality ladder.[12] In that sense, in the model, the comparative advantage always refers to particular goods, irrespective of the quality at which those goods are actually produced (for example, a country that has a comparative advantage in producing fruit products, will have this advantage both in organic and in non-organic fruit products).

From the international pricing equation (1) and Assumption 1, we can derive the marginal good m (that is, the good that can be supplied by both countries at the same price), which satisfies:

$$A(m) = w/w^*. \tag{7}$$

Equation (7) implies that, *given* the relative wage w/w^*, H will produce all the goods in the interval $[0, m]$ and F will produce all the goods within $[m, 1]$.

In order to allow countries to possibly display identical income per head in equilibrium (that is, in order to remove any direct source of *absolute* advantage from the model), we pose the next assumption, which formally states symmetry in terms of countries' comparative advantage.

Assumption 2 (Symmetric comparative advantages) *We suppose:* $v_0 = 0.5$.

[12] Letting $\eta(\cdot)$ vary across countries change in a similar fashion as $a(\cdot)$ would not qualitatively alter the results of the paper —in fact, adding heterogeneity on $\eta(\cdot)$, on top of that on $a(\cdot)$, would *reinforce* our findings.

Additionally, to disregard the effects of heterogeneous population size in different countries, we initially suppose that both H and F are inhabited by a continuum of individuals with identical mass, which we normalise to one. (We explore the general equilibrium effects of heterogenous population size later on in Section 4.2.)

A representative individual from H will then solve:

$$\begin{aligned} &\max_{\{q_v,\beta_v\}_{v\in\mathbb{V}}} \quad U=\int_0^m q_v \ln\left(\frac{\beta_v\kappa}{a\left(v\right)q_v^{\eta(v)}}\right)dv+\int_m^1 q_v \ln\left(\frac{\beta_v\kappa\, w}{a^*\left(v\right)q_v^{\eta(v)}}\right)dv, \\ &\text{subject to:} \quad \int_{\mathbb{V}}\beta_v\, dv=1;\ \text{and}\ q_v\geq 1,\ \forall v\in\mathbb{V}. \end{aligned} \tag{8}$$

On the other hand, a representative individual from F solves:

$$\begin{aligned} &\max_{\{q_v^*,\beta_v^*\}_{v\in\mathbb{V}}} \quad U^*=\int_0^m q_v^* \ln\left(\frac{\beta_v^*\kappa}{a\left(v\right)q_v^{\eta(v)}w}\right)dv+\int_m^1 q_v^* \ln\left(\frac{\beta_v^*\kappa}{a^*\left(v\right)q_v^{\eta(v)}}\right)dv, \\ &\text{subject to:} \quad \int_{\mathbb{V}}\beta_v^*\, dv=1;\ \text{and}\ q_v^*\geq 1,\ \forall v\in\mathbb{V}. \end{aligned} \tag{9}$$

The solution of (8) and (9) yields the demand functions of each good $v\in\mathbb{V}$ by H and F, respectively. By using $\vartheta\left(v\right)\equiv\int_0^v\beta_z\,dz$ (as defined in Corollary 1) and $\vartheta^*\left(v\right)\equiv\int_0^v\beta_z^*dz$ (see Corollary 2 in Appendix B), we can write the equilibrium condition for the market of goods produced in H as follows:

$$\vartheta\left(m\right)w+\vartheta^*\left(m\right)=w, \tag{10}$$

where m is the marginal good as defined by (7). Condition (10) essentially says that the aggregate amount of income spent by the world in goods produced in H must be equal to the aggregate income of H. This condition can also be understood as the equilibrium condition for the labour market in H.[13]

The world economy general equilibrium is determined by (7), (8), (9), and (10). Henceforth, we will focus our attention on the equilibrium values of w and m, and on how these two variables respond to some comparative statics experiments commonly explored by the previous literature on international trade with nonhomothetic preferences. Firstly, we analyse the general equilibrium consequences of uniform aggregate productivity growth in the world economy; this exercise shows how our model can account for income divergence across countries with similar initial conditions purely via the endogenous evolution of the terms of trade. Secondly, we investigate the

[13]Because of the Walras' Law, an analogous condition can be derived for the equilibrium in the labour market in F.

effects of uneven population growth across countries, and illustrate how the country in which population grows faster tends to experience a decline in its terms of trade and relative income. Lastly, we look at the case of income inequality within countries, and discuss how inequality tends to improve the terms of trade and the relative income of the economy that specialises in producing goods that display higher income demand elasticity, regardless of whether inequality arises in H or F.

4.1 Worldwide Uniform Aggregate Productivity Growth

In this subsection, we look at the impact of changes in κ on the equilibrium values of w and m. We can split the results in two different cases.

Subsistence economies: $\kappa \leq \underline{\kappa}$

From our previous discussion, we can observe that when $\kappa \leq \underline{\kappa}$, the optimal demand intensities are set at $\beta_v = \beta_v^* = 1$ for all $v \in \mathbb{V}$. This result in turn implies that $\vartheta(m) = \vartheta^*(m) = m$. Therefore, (10) simplifies to:

$$w = m/(1-m). \tag{11}$$

Combining (7) with (11), leads to $m/(1-m) = A(m)$, from where it follows that, for all $\kappa \leq \underline{\kappa}$: $w = 1$ and $m = 0.5$. That is, H and F exhibit the *same* level of income, and the pattern of regional specialisation is accordingly dictated by the "natural" comparative advantage of each country without the relative-wage effect (i.e., those that derive purely from Assumption 1).[14]

Modern economies: $\kappa > \underline{\kappa}$

When aggregate productivity is sufficiently high, the income equality between H and F no longer holds. In particular, as κ rises above the threshold $\underline{\kappa}$, the terms of trade start moving in favour of H, and thus H becomes relatively richer than F. Moreover, the income disparity between H and F further increases as κ keeps rising.

Proposition 2

Let $\kappa > \underline{\kappa}$. Then, in equilibrium:

[14] Notice that, since $w = 1$ for all $\kappa \leq \underline{\kappa}$, in fact $\underline{\kappa} = \underline{\kappa}^*$ (that is, the threshold on κ that divides a subsistence-economy from a modern economy happens to be the same for both H and F). As a consequence, we can refer to *both* thresholds simply as $\underline{\kappa}$.

(i) $w > 1$; $m < 0.5$;

(ii) $\partial w/\partial\kappa > 0$; $\partial m/\partial\kappa < 0$.

Proof.

Part (i). When $\kappa > \underline{\kappa}$, from Corollary 1 and 2 it follows that $\vartheta(m) > m$ and $\vartheta^*(m) > m$. As a result, by using (10), we can obtain:

$$w = \frac{\vartheta^*(m)}{1-\vartheta(m)} > \frac{m}{1-m}. \tag{12}$$

Combining next (12) with (7), and recalling Assumption 1 and 2 leads to:

$$A(m) = \frac{\vartheta^*(m)}{1-\vartheta(m)} > \frac{m}{1-m} \quad \Leftrightarrow \quad m < 0.5.$$

Finally, since $m < 0.5$, equation (7) implies that $w > 1$.

Part (ii). Next, to study how w and m vary as κ keeps rising above $\underline{\kappa}$, we differentiate the equilibrium conditions (7) and (10). This leads to:

$$\frac{\partial w}{\partial\kappa} = A'(m)\frac{\partial m}{\partial\kappa} \tag{13}$$

and

$$(w\beta_m + \beta_m^*)\frac{\partial m}{\partial\kappa} + \left(w\frac{\partial\vartheta(m)}{\partial w} + \vartheta(m) + \frac{\partial\vartheta^*(m)}{\partial w}\right)\frac{\partial w}{\partial\kappa} + \left(\frac{\partial\vartheta(m)}{\partial\kappa} + \frac{\partial\vartheta^*(m)}{\partial\kappa}\right) = \frac{\partial w}{\partial\kappa}, \tag{14}$$

where the first term in (14) uses the fact that $\partial\vartheta(m)/\partial m = \beta_m$ and $\partial\vartheta^*(m)/\partial m = \beta_m^*$. Plugging (13) into (14), we can obtain:

$$\frac{\partial m}{\partial\kappa} = \frac{\partial\vartheta(m)/\partial\kappa + \partial\vartheta^*(m)/\partial\kappa}{[1-\vartheta(m) - w\,\partial\vartheta(m)/\partial w - \partial\vartheta^*(m)/\partial w]\,A'(m) - (w\beta_m + \beta_m^*)}. \tag{15}$$

For determining the sign of (15), we can use the following two results: first, Corollary 1 states that both $\partial\vartheta(m)/\partial\kappa > 0$ and $\partial\vartheta^*(m)/\partial\kappa > 0$; second, as shown in Appendix D, $\partial\vartheta(m)/\partial w \leq 0$ and $\partial\vartheta^*(m)/\partial w < 0$. Therefore, since $1 - \vartheta(m) > 0$ and $A'(m) < 0$, then $\partial m/\partial\kappa < 0$ obtains from the right-hand side of (15). Finally, from (13) it then follows that $\partial w/\partial\kappa > 0$. ■

Proposition 2 shows that as the world aggregate-productivity parameter, κ, increases, the income in H eventually begins diverging away from the income in F. The reason for the divergence rests on the fact that H enjoys a comparative advantage in producing lower-indexed goods, which tend to be consumed in relatively higher qualities and display accordingly higher income demand elasticity. As a consequence, as the world productivity grows *uniformly* above $\underline{\kappa}$, aggregate world expenditure shifts towards the set of commodities produced by H. The ensuing excess demand for commodities produced in H causes excess labour demand in H and w thus goes up. In turn, as w rises, the marginal good moves to the left (i.e., m falls), and some of the goods that used to be produced by H start being produced by F, restoring the equilibrium in the labour markets.

The endogenous emergence of income disparities in the absence of absolute advantage in this two-country world economy represents the main result and novelty of our paper. Initially, H and F display the same level of income per head. Although sectorial productivities do differ across the two countries and govern the patterns of regional specialisation, this heterogeneity does not prove enough to warrant income disparities between H and F. This is because at low levels of worldwide aggregate productivity the willingness to pay for high-quality commodities is not large enough for tilting aggregate demand disproportionately towards the goods produced by H. However, in a context where worldwide aggregate productivity rises, leading to higher incomes in both H and F, goods exhibiting larger scope for quality upgrading become increasingly appreciated by the consumers and, thus, start absorbing larger budget shares. Within a general equilibrium framework, this mechanism implies that aggregate demand shifts towards H, inducing faster income growth in H relative to F, via the secular tendency to improve H's terms of trade.

Concerning the evolution of the world productive structure, the equilibrium adjustments triggered by worldwide uniform productivity growth generates two types of product cycle phenomena. First, the marginal good, m, shifts left: this is an *international product cycle* phenomenon involving both countries simultaneously, similar to that previously discussed by Linder (1961) and Vernon (1966), where over time the production of lower-quality goods moves from H to F, while H specialises in more sophisticated higher-quality goods. The second phenomenon occurs within each good and could be denoted *regional product cycle*, as it involves single countries individually: rising world income leads both H and F to abandon the manufacturing of lower-quality goods and replace them with the production of goods of higher quality (i.e., the optimal q_v tends to rise for all goods traded in the world economy as κ increases).

An immediate implication of the regional product cycle phenomenon is the fact that citizens from H, who are richer than those from F, consume comparatively higher qualities for each good traded in the international markets, i.e. $q_v \geq q_v^*$, $\forall v \in \mathbb{V}$. This result is consistent with various strands of empirical evidence. For instance, Verhoogen (2008) and Iacovone and Javorcik (2009) show that Mexican manufacturing plants produce higher-quality versions of goods to export to richer markets (mainly the US). Similar evidence is provided by Brooks (2006) for Colombian manufacturing plants. A more general piece of evidence comes from Hallak and Schott (2009) who using cross-country data show that the quality gap in production between rich and poor economies is smaller than their income gap, which suggests that poorer economies are producing high-quality goods to sell in richer markets. The same conclusion follows from Fieler (2007) who reports that unit prices (a proxy for quality) rise with the importer's income per capita, even for goods originating from the same exporter and commodity category.

As a final remark, notice that the proof of result (*ii*) in Proposition 2, which states that wages rise and the marginal good shifts leftwards as world aggregate-productivity increases, does not rely on Assumption 2 at any moment. In fact, both $\partial w/\partial \kappa > 0$ and $\partial m/\partial \kappa < 0$ would still obtain if we instead gave a larger range of initial advantage to F by assuming that $\upsilon_0 < 0.5$. If $\upsilon_0 < 0.5$, somewhat richer dynamics would be obtained, though. More precisely, $w < 1$ would hold for levels of worldwide productivity below a certain (finite) threshold $\widehat{\kappa} > \underline{\kappa}$, while the model would predict catching-up by H for values of $\kappa \in (\underline{\kappa}, \widehat{\kappa})$, followed next by overtaking and divergence in an analogous fashion as it occurred when $\upsilon_0 = 0.5$.

4.2 Uneven Population Growth

In this subsection we let the population size in F differ from that in H. In particular, we let the total mass of individuals in F equal $L > 1$, while we keep the total mass of individuals in H equal to 1. Thus, the labour market equilibrium condition in H will be given by:

$$\vartheta(m)w + L\vartheta^*(m) = w. \tag{16}$$

Visual inspection on (16) and (10), combined with (7), immediately implies that the equilibrium value of w that is delivered by (16) will be strictly larger than that yielded by (10). In particular, in equilibrium $w > 1$, regardless of the value of κ. Furthermore, this source of income disparity between F and H magnifies as the value of L rises. This is because, when the population in F increases, the relative wage w must go up so as to accommodate the excess supply of labour in F. More precisely, a larger L requires more goods to be produced by F in order to keep full employment there; this is accomplished by letting w go up, which in turn shifts the marginal good m to the left, helping restore the equilibrium in the labour markets.

The result that, as the relative population of F increases, the H relative wage rises is in line with the models in Flam and Helpman (1987), Stokey (1991) and Matsuyama (2000). However, some interesting differences are also present. In Flam-Helpman and Stokey, although the optimal bundle of goods traded in the market changes, no *new* goods actually appear in the world economy as w rises due to uneven population growth in the world. In Matsuyama, new goods start being produced, but this happens *only* in the country whose population grows slower (i.e., in H); the country whose population grows faster, F, does not introduce new goods into the world markets, but only takes on the production of (some) goods that are abandoned by H as w increases. In our model, new goods actually start being produced by F as its relative wage decreases owing to faster population growth. A higher w brings about two different effects: first, individuals in H become richer (income effect); second, the relative prices of the goods originally produced in F decline (substitution effect). Taken jointly, these two effects reinforce one another and induce individuals from H to start demanding higher qualities for the goods produced in F.[15]

4.3 Income Inequality within Countries

In this subsection we discuss the general equilibrium consequences of introducing some degree of income heterogeneity within countries. Here, analogous qualitative results are generated regardless of whether inequality is introduced in F or H (or in both at the same time). Therefore, for brevity, in what follows we focus only on the first case.

Assume that F is inhabited by two types of individuals: p and r, where the p stands for *poor* and r stands for *rich*. Each sub-group of individuals from F has mass equal to 0.5. The difference between the two sub-groups lies in that a type p is endowed with a smaller amount of effective labour than a type r. In particular, suppose that type-p individuals are endowed with $1 - \iota$ units of effective labour and individuals in r are endowed with $1 + \iota$ units of it, where $\iota \in (0, 1)$. On the other hand, in H everyone is endowed with the same amount of effective labour. Introducing income inequality in the model leads to interesting results when the types p are so poor that, in equilibrium, they consume all goods at the baseline quality level, whereas in contrast the types r can afford consuming at least some of the goods strictly above that level. To focus on such case, we accordingly set $\kappa = \underline{\kappa}$.

Introducing income inequality in F raises the relative wage in H. This is owing to the nonhomotheticity of the demand schedules of the rich foreigners. More precisely, increasing ι transfers income from the poor foreigners, who spend a fraction m of it in goods from H, to the rich foreigners, who spend a fraction $\vartheta_r^*(m) > m$ of their income on those commodities. As a result, aggregate demand for goods produced in H rises, leading to higher w. Similarly, it is quite

[15] For example, our model then predicts that Africa will start to produce, say, organic bananas to sell in Europe, as increasingly richer European consumers begin desiring to purchase higher-quality fruit products, which are moreover becoming relatively cheaper over time as population in Africa grows faster than in Europe.

straightforward to observe that incorporating inequality in H would carry similar consequences on w and m. This is the case because the rich locals would tend to shift demand towards the goods produced in H; exactly the same shift induced before by the presence of rich foreigners.

5 Length of Quality Ladders and Exports Behaviour: a brief examination of the trade data

In this section, we conduct a few empirical exercises to assess whether the central predictions of our model find some empirical support in the trade data. More specifically, our aim is to determine whether the exports, both at a world- and country-level of aggregation, correlate with world income changes in a way that is consistent with our previous theoretical findings. For this purpose, we propose two stylised reduced-form approaches to assess our predictions. The first approach, illustrated in Section 5.2, looks at whether long-ladder goods attract increasing world expenditure shares with rising world income. The second approach, illustrated in Section 5.3, investigates whether exports from countries specialising in the production of long-ladder goods increase relatively more when world income rises. Furthermore, we look at how economies' initial specialisation correlates with future exports growth in a context of world income growth, in the attempt to investigate whether the initial pattern of specialisation may lead to uneven future exports growth when goods differ in their scope for quality upgrading.

5.1 The Length of Quality Ladders Across Goods

The first step is to construct a proxy for the length of quality ladders across a large set of different types of tradeable goods. We build two different proxies based on measures of dispersion of import unit prices using the data compiled by Feenstra *et al* (2005). This dataset documents bilateral trade at the country level for the period 1962-2000 measured both in value and quantities, organised following the 4-digit Standard International Trade Classification (SITC-4), Revision 2. From this dataset, we calculate the (average) import unit prices of each SITC-4 product by each importer during the year 2000.[16] As a result, we are able to obtain up to 182 different unit

[16] We choose to calculate unit prices using *only* the year 2000 for two different reasons. First, it avoids problems that may arise from comparing unit prices at different points in time. Second, and more importantly, using the last year available seems the most promising one in terms of proxying the length of ladders according to the nature of nonhomotheticities in our model. This is because the poorest country in 2000 was roughly as poor as the poorest one in 1962, whereas the richest economy in 2000 was substantially richer than the richest one in

prices (one for each importer) for each of the 749 different goods in the SITC-4 categorisation.

In order to construct our first proxy for the length of quality ladders, we sort the (average) import unit prices obtained for every importer, and pick the maximum and minimum for each of the 749 goods in the SITC-4 categorisation. We next use these two boundary prices to compute the max-to-min price ratio for every SITC-4 good. In that regard, unit prices of each SITC-4 good are taken as proxies for the intrinsic qualities of that particular good, and the max-to-min price ratios are accordingly viewed as proxies for the length of quality ladders of goods.[17]

To obtain our second proxy for the length of quality ladders, we compute the coefficients of variation of the distribution of unit prices for each of the SITC-4 goods.[18] The underlying idea for this measure is that goods featuring longer quality ladders should, in general, also display a more 'dispersed' distribution of unit prices.[19]

In Table 1 we group all the SITC-4 sectors/goods into their corresponding 1-digit sector. Therein we report the average values of the max-to-min unit price ratios and the average values of coefficients of variation of unit prices. With the exception of sector 2, Table 1 seems to point to the common perception that the quality ladders of primary goods tend to be shorter than those of manufacturing products (i.e. sectors 5 to 8). Sector 9, which contains only 5 products in the SITC-4 classification, appears to be an outlier when compared to the other sectors.

1962. (In 1962 the poorest and richest economies were Guinnea Bissau and Switzerland with GDP per head 417 and 19512, respectively, measured in PPP 2005 US dollars. In 2000, the poorest was Zaire with GDP per capita equal to 312 and the richest was Luxemburg with 63419, both measured in PPP 2005 US dollars.)

[17] In order to mitigate the effect of outliers and measurement errors, when computing the max-to-min price ratios we clean the data on unit prices along two different dimensions: *i*) we disregard unit prices from importers whose reported import quantities of those particular SITC-4 goods equals 1; *ii*) we disregard the lowest price when the second-lowest recorded unit price is more than 100% larger than the former and, similarly, we disregard the highest price if this one more than doubles the second-highest recorded unit price (when this occurs, we utilise the second-lowest or second-highest unit prices to build the extreme price ratios).

[18] In this case, we also clean the data of import unit prices following the same two procedures stated in the previous footnote.

[19] As mentioned previously in the Introduction, unit prices/values have been used before as proxies of quality in the empirical trade literature: e.g., Schott (2004), Hallak (2006), Fieler (2007). Of course, unit prices/values should only be taken as an imperfect proxy of the intrinsic quality of the commodity, since factors other than quality may also be affecting unit prices (for example, the degree of horizontal differentiation across industries, heterogeneous transport costs, trade tariffs).

Table 1: Averages at 1-digit level of disaggregation

SITC-1 Sector	Number of products in SITC-4 classif.	Average of max-to-min unit price ratios	Average of coeff. of variation of unit prices
0 - Food and live animals	93	46.7	0.711
1 - Beverages and tobacco	11	25.1	0.656
2 - Crude materials, inedible, except fuels	101	134.6	1.084
3 - Mineral fuels, lubricants and rel. materials	20	44.6	0.876
4 - Animal and vegetable oils, fats and waxes	18	11.8	0.526
5 - Chemicals and related products	91	177.8	1.024
6 - Manufactured goods classified chiefly by material	175	102.7	0.952
7 - Machinery and transport equipment	157	186.1	0.919
8 - Miscellaneous manufactured articles	78	100.8	0.887
9 - Commodities and trans. not classified elsewhere	5	1450.7	2.309
ALL GOODS	**749**	**130.6**	**0.927**

5.2 Cross-Good Regressions

One of the main predictions of our model is that goods with lower cost of quality upgrading tend to feature longer quality ladders and exhibit higher income demand elasticities. As a result, long-ladder goods will tend to attract increasing world expenditure shares with rising world income. We assess this prediction on a reduced-form approach by running the following regression:

$$\Delta X_{j,t} = \alpha + \beta\left(\Delta Y_{w,t} \times Ladder_j\right) + \eta_t + \varepsilon_{j,t}; \tag{17}$$

where $\Delta X_{j,t}$ is the percentage growth of the total value of world exports (and imports) of good j in year t, $\Delta Y_{w,t}$ is the percentage growth of world income per head in year t, and the variable $Ladder_j$ denotes the length of the quality ladder of good j. Our model thus predicts $\beta > 0$ because goods with larger scope for quality upgrading should also display higher income demand elasticities. Notice that since (17) includes year fixed effects (η_t), we do not need to include $\Delta Y_{w,t}$ as another independent variable because such effect will be fully captured by each η_t (more precisely, $\Delta Y_{w,t}$ and η_t are perfectly colinear).

Table 2 displays the results of the cross-good regressions. The regressions using 'interaction term version 1' are run with the length of ladder being proxied by the max-to-min unit price ratios. In the first column we use all the 4-digit goods. Column (2) removes petroleum related goods (all goods coded 3300 through to 3400 in the dataset) in case these goods have a particular influence on the results for reasons other than nonhomotheticity linked to heterogeneity in the scope of quality differentiation (for example, petroleum related goods may display relatively high income demand elasticity not necessarily because they exhibit long quality ladders, but because they may represent fundamental inputs in the production of long-ladder goods). Column (3) removes all five goods in 'Commodities and transactions not classified elsewhere' as they seem to be clear outliers according to Table 1. In the three cases, results are consistent with our model's prediction and all the estimates of β are significantly different from zero at 10% level. Similar conclusions (at somewhat higher significance levels) follow from regressions in columns (4)-(6), where the length of ladder is proxied by the coefficients of variation of unit prices.[20]

Finally, in columns (7) and (8) we run regression (17) using only sectors producing primary goods (excluding again petroleum related goods). Column (7) seems to suggest that the link between scope of quality upgrading (proxied by the max-to-min price ratios) and income demand elasticity is also present when we look only at primary goods; in (8) the coefficient is also positive although it fails to reach significance at 10% level. This last result may suggest that our mechanism might be relevant not only when comparing manufacturing versus agricultural goods, as the Prebisch-Singer hypothesis has been traditionally presented in the North-South literature. In particular, our model may also apply to cases in which the patterns of specialisation are not strictly linked to different stages of economic development.[21]

5.3 Cross-Country Regressions

Our model also predicts that countries that specialise in the production of goods with longer quality ladders should see their exports value increase more strongly when world income rises. We assess this prediction resorting again to a reduced-form approach by conducting the following regression:

$$\Delta X_{i,t} = \delta + \gamma\left(\Delta Y_{w,t} \times Ladder_{i,t}\right) + \mu_t + \nu_i + \upsilon_{i,t}; \tag{18}$$

[20] Notice that since (17) is capturing a reduced-form correlation between the variables placed in the regression, the product $(\beta \times Ladder_j)$ cannot actually be interpreted as a component of the income demand elasticity of good j. More precisely, to estimate such elasticity we would first need to abstract from all general equilibrium interactions that take place when international prices adjust to income shocks (in terms of our general equilibrium model, we would need to remove the effects brought about by the changes of w resulting from variations in κ). Nevertheless, our general equilibrium model still predicts a positive correlation between changes in the value of exports of good j and its ladder's length in a context of positive world income growth, which is the correlation we aim to capture with (17).

[21] An illustrative example of how our model can be applied to rationalise patterns of specialisation of countries at similar stages of economic development is presented next in Section 6, where we discuss the case of colonial Jamaica and compare it to the one of pre-industrial Argentina.

where $\Delta X_{i,t}$ is the percentage growth of the total value of exports by country i in year t. The variable $Ladder_{i,t}$ measures the average length of ladders of the bundle of goods exported by i in year t: we build this variable by weighing $Ladder_j$ (i.e., the variable used before in our cross-good regressions) of each good j by the share of that good in the total value of exports of country i in period t. Our model thus predicts $\gamma > 0$.

Columns (1)-(5) in Table 3 show the results of different versions of (18) when we measure the length of ladders by the max-to-min unit price ratios. Column (1) and (2) use all countries in the panel, the former including country fixed effects and the latter excluding them: in both cases the estimates are positive, similar in magnitude, and highly significant. In (3) we exclude countries from the OPEC from the regression in case oil exporters may have a large impact on the results (in particular, given that our sample includes years when the oil shocks occurred); the previous results remain essentially intact. Results also remain unchanged when we exclude Latin American economies from the sample in column (4) – in this case, the rationale is the fact that many of these economies have gone through severe macroeconomic and external crises during '80s and '90s, including large devaluations of their currencies. Finally, in (5) we exclude the OECD countries to have some feeling about whether our results are crucially driven by comparing developed economies to less developed ones; as we can readily observe, results still remain essentially unaffected when we restrict the sample in such a way.

In columns (6)-(10) we replicate the same regressions using the coefficients of variation of unit prices. Although the significance of the estimates is lower than in (1)-(5), and in (6) and (7) we fail to reach significance at 10% level, all the estimates carry the expected sign and, moreover, their magnitudes exhibit a similar pattern to those in (1)-(5).

Viewed from a longer run perspective, our model argues that, if countries' comparative advantages across goods remain constant over time, the initial pattern of specialisation may lead to uneven future exports growth when goods differ in their scope for quality upgrading. Table 4 looks at how economies' initial specialisation correlates with future exports growth in a context of positive world income growth. In particular, we are interested in investigating whether, when focusing on periods of positive world income growth, countries that initially specialise in goods with longer quality ladders will tend to experience a higher rate of growth of their exports. For this purpose, we build a proxy for country i initial specialisation –in terms of scope for (future) quality upgrading– by weighting the length of ladders of SITC-4 products

Table 2: Cross-good panel regressions

	Dependent Variable: rate of growth of world exports (and imports) of SITC-4 products							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Interaction term version 1 (ΔY_w × Ladder length)	0.115	0.111	0.144				0.566	
	(1.67)*	(1.64)*	(1.71)*				(2.23)**	
Interaction term version 2 (ΔY_w × Ladder length)				0.139	0.139	0.138		0.129
				(1.96)**	(1.95)**	(1.82)*		(1.29)
Includes Petroleum related goods	YES	NO	NO	YES	NO	NO	-	-
Includes sector 9 of SITC-1	YES	YES	NO	YES	YES	NO	-	-
Includes only SITC-1 sectors 0 to 4 (excl. petroleum)	-	-	-	-	-	-	YES	YES
Observations	22485	22186	22058	22642	22163	22035	7488	7465
Number of SITC-4 goods	749	739	734	748	738	733	233	232
R - Squared	0.23	0.23	0.23	0.23	0.23	0.23	0.21	0.21

Robust absolute t-statistics clustered at the SITC-4 product level in parentheses. All data is for years 1963-2000. All regressions include year dummies and a constant term.
Interaction term version 1 uses the ratio of maximum to minimum unit prices (divided by 1000) to proxy for length of ladder of SITC-4 products.
Interaction term version 2 uses the coefficient of variation of unit prices to proxy for length of ladder of SITC-4 products. * significant at 10%; ** significant at 5%; *** significant at 1%.

Table 3: Cross-country panel regressions

	Dependent Variable: rate of growth of total exports of each country									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Interaction term version 1 (Y_w × Ladder length)	1.392	1.294	1.515	1.518	1.495					
	(2.46)**	(3.94)***	(2.70)***	(2.55)***	(2.40)**					
Interaction term version 2 (Y_w × Ladder length)						0.844	0.691	1.199	1.306	1.185
						(1.35)	(1.56)	(1.85)*	(1.92)**	(1.65)*
Country fixed effects	YES	NO	YES	YES	YES	YES	NO	YES	YES	YES
Excludes OPEC countries	NO	NO	YES	YES	YES	NO	NO	YES	YES	YES
Excludes Latin American countries	NO	NO	NO	YES	YES	NO	NO	NO	YES	YES
Excludes OECD countries	NO	NO	NO	NO	YES	NO	NO	NO	NO	YES
Observations	5458	5458	5070	4511	3580	5458	5458	5070	4511	3580
Number of countries	182	182	170	155	127	182	182	170	155	127
R - Squared	0.20	0.19	0.21	0.19	0.17	0.20	0.19	0.21	0.19	0.16

Robust absolute t-statistics clustered at the country level in parentheses. All data is for years 1963-2000. All regressions include year dummies and a constant term.
Interaction term version 1 uses the ratio of maximum to minimum unit prices (divided by 1000) to proxy for length of ladder of SITC-4 products.
Interaction term version 2 uses the coefficient of variation of unit prices to proxy for length of ladder of SITC-4 products. * significant at 10%; ** significant at 5%; *** significant at 1%.

Table 4: Future exports growth regressions

	Average growth of exports using years of positive world income growth Sample: years 1971-2000							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Initial ladder length (1962-66) version 1	3.91	4.58	5.22	5.21				
	(2.48)**	(2.70)***	(2.50)**	(2.42)**				
Initial ladder length (1962-66) version 2					1.40	2.87	2.80	2.50
					(1.18)	(2.36)**	(2.22)**	(2.05)**
Excludes OPEC countries	NO	YES	YES	YES	NO	YES	YES	YES
Excludes Latin American countries	NO	NO	YES	YES	NO	NO	YES	YES
Excludes OECD countries	NO	NO	NO	YES	NO	NO	NO	YES
Observations	140	129	114	89	140	129	114	89
R - Squared	0.04	0.06	0.07	0.07	0.01	0.04	0.04	0.03

Initial ladder length version 1 uses the max-to-min unit price ratios (divided by 1000) to proxy for length of ladder of SITC-4 products, weighting these ratios by the average exports shares of each product during years 1962-66. Initial ladder length version 2 uses the coeffients of variation of unit prices. Dependent variable is the average growth of total exports by each country during years in the sample in which world income per head displays positive growth. Robust t-statistics in parentheses. All regressions include a constant term. * significant at 10%; ** significant at 5%; *** significant at 1%.

by their average export shares during years 1962-66. Notice that, although we use the first five years in the sample for the export-share weights, the length of ladders for each SITC-4 good are still being proxied by the max-to-min price during the last year in the sample (i.e. year 2000). This is in line with the logic of our model, where goods' *potential* of quality upgrading is the feature that really matters for long-run exports growth, and the process of quality upgrading itself materialises over time as world productivity expands and world income rises accordingly.

In Table 4 we take the subsample of years in the period 1971-2000, and we compute the average growth of exports of each country, using only years in which world income per head growth was positive. Next, we regress those average growth rates on countries' initial length of ladder, which are built as explained in the above paragraph. As for Table 3, we run regressions including all countries in the sample, and subsequently excluding OPEC, Latin American and OECD countries, in that order. All the regressions (except for column 5) reach the same conclusion: the patterns of specialisation during the first five years in the sample (in terms of average length of ladders of exports during 1962-66) correlates significantly with the average growth of future exports during years of positive world income growth. (We ran the same regressions using the subsample 1981-2000; the results obtained are qualitatively similar to those in Table 4 and are available from the authors upon request.)[22]

[22] All regressions in Table 3 and Table 4 include sector 9 of the 1-digit SITC categorisation. Results obtained excluding goods in sector 9 are very similar to those presented before, and are available upon request.

6 An Illustrative Historical Example: colonial Jamaica and pre-industrial Argentina

Situations where the mechanism proposed in this paper may have played an important role include the cases of economies for which exogenous initial geographical conditions greatly influenced their specialisation in the world economy during some period in history. As an illustrative example, we take the case of colonial Jamaica (denoted by J) and compare it to the one of pre-industrial Argentina (denoted by A). In this example, we consider two goods, namely sugar (denoted by s) and beef (denoted by b).

From the second half of the XVII century until the first half of the XIX century, the Jamaican economy grew mainly based on the production and export of sugar from sugarcane. This is not surprising given the excellent climatic conditions this tropical island offered for that type of crop. By 1805, Jamaica was the largest sugar exporter in the world (Higman, 2005). Given the value attributed to sugar by European consumers, during that period Jamaica was deemed probably the most important British colony in the Americas (Hall, 1959; Sheridan, 1973). Although sugar was indeed a very valuable consumption good at that time, it clearly was a type of good with very limited scope for undergoing subsequent improvements in quality. As such, according to our model, sugar was bound to eventually lose its status of luxury among consumers as their incomes would rise.[23] In fact, by the second half of the XIX century, sugar began to lose its economic preeminence in the world markets and started experiencing a long phase of declining (relative) prices, which in turn seriously damaged the Jamaican economy.[24]

In Argentina, geographical conditions made this country exceptionally apt for the breeding of cattle and growing cereals, which constituted the main engines of its economy until 1914. The commercial production of cattle started in the late second half of the XVIII century with the appearance of the *saladeros* – slaughterhouses where meat would be cured by drying and salting (Newton, 1966). Salt-cured beef was a rather unsophisticated product that was mostly

[23] In that regard, Sheridan (1973) writes, "Until the late years of the 17th century English sugar consumption seems to have been confined rather closely to the wealthy sections of society. [...] Lower income groups were reported to have used quantities of molasses, treacle and low-quality sugar to sweeten their eatables, and to make drinkable liquors. [...] During the 18th century the [*physical*] demand for sugar grew so rapidly among all sections of English society that few people considered it a luxury [*anymore*]."

[24] By 1820 the GDP per head in Jamaica was 1.05 times the world average GDP per head, whereas by 1870 it equalled 0.61 the world average GDP per head. In fact, during the period 1820-1870, income per head in Jamaica fell 23.6%, while the world average income per head grew 31%. Data taken from Maddison (2008).

exported to Cuba and Brazil to feed slaves. In fact, the industry of the *saladeros* did not mean a big push to the Argentinean economy, which was at that time still a very marginal country within the world economy.

The *big boom* for the cattle industry in Argentina came much later, at the end of the XIX century. Unlike the sugar industry, the cattle industry had some scope for quality upgrading, in the form of chilled and frozen beef. The market size for this product, certainly more appreciated by consumers than salt-cured beef, was however initially quite limited, since the transportation cost induced a huge differential in the prices of the two goods. Yet, in Europe, incomes had been continuously rising during the XIX century, thanks to the massive technological advancement that followed the advent of the Industrial Revolution. The availability of a higher-quality commodity in the cattle industry eventually attracted well-to-do European consumers, whose demand induced Argentinean firms to export large amounts of chilled and frozen beef to Europe.[25] During the period 1890-1914, Argentina grew on average at rate of 5.5% yearly, attracted millions of immigrants from Europe and became one of the richest countries in the world (Maddison, 2008).[26] The exportation of chilled and frozen beef was undoubtedly one of the main activities that spurred this phase of fast and steady economic growth in Argentina between 1880-1914 (Rapaport, 1988).

This example illustrates how exogenous geographical conditions greatly influenced the path of GDP growth in Jamaica and in Argentina via the evolution of their exports, in the way our model would predict. Jamaica was comparatively efficient at producing sugar, while Argentina enjoyed a comparative advantage in beef production (in terms or our model, $a_s^J < a_s^A$ and $a_b^A < a_b^J$). Sugar offered very limited scope for quality improvements, which is analogous to assuming that the cost of quality upgrading for sugar products is extremely high. On the contrary, beef did offer some more scope for quality upgrading than sugar (in our model, $\eta_b < \eta_s$). The latter materialised in the switch from salt-cured beef production (lower-quality commodity) to chilled and frozen beef (higher-quality commodity). As predicted by our model, sugar exports initially sustained high growth in Jamaica, until rising income in the world (i.e., driven by rising κ) shifted aggregate world expenditure towards goods which could be offered in higher quality

[25] The main market for Argentinean chilled and frozen beef at that time was by far the prosperous Great Britain of end of XIX and beginning of XX century (in 1914, 83.5% of the total Argentinean exports of chilled and frozen beef was sent to the UK). See Rapaport (1988).

[26] By 1913, the GDP per head in Argentina was slightly larger than that of France and Germany, and it was 0.77 times the GDP per head in the UK. Data taken from Maddison (2008).

degrees, such as chilled and frozen beef from Argentina.[27]

7 Conclusion

We have proposed a model of international trade with nonhomothetic preferences based on comparative advantages that are *unrelated* to the stage in the process of development in which countries are. This feature represents the main point of departure with respect to the past literature on North-South trade, where the comparative advantage stems from the fact that some countries (the North) have historically accumulated larger amounts of capital than others (the South).

The key novel finding of our model is pointing out that even when no single country enjoys a clear *absolute* advantage over any other country and productivity changes are uniform and identical in all countries, international trade may still be the source of income divergence in the world economy when nonhomothetic preferences and quality ladders are jointly taken into account. In particular, countries' incomes will diverge when comparative advantage induce patterns of specialisation that, although optimal for each country at early stages in the process of development, do not offer the same scope for improvements in terms of quality upgrading of final products in the long run.

Our model also points out that worldwide uniform productivity growth generates two distinct types of product cycle phenomena. The first is an *international product cycle* phenomenon *à la* Linder-Vernon, where over time one economy takes on the production of goods previously produced by another economy. The second – which is novel to our model – is a *regional product cycle* that occurs within each good and within each economy: rising world income makes *all* economies engage in the production of newer goods of higher quality (so as to satisfy the increasing demand for high qualities by wealthier consumers).

Admittedly, we have presented a largely stylised model, simplified in several dimensions, so as to illustrate in a concise way our main mechanism and its implications: the fact that the dy-

[27] One may argue that the productivity improvement discussed in the illustrative example should be read as a rise in good-specific productivity (in our model, a fall in a_b) rather than an increase in aggregate productivity. Even if we supposed that was the case, a mere change in this parameter would not necessarily rationalise the facts illustrated in this example, as the way total expenditure shares in good b would respond to a change in a_b crucially depends on the price elasticity of beef demand. Therefore, variations in a_b would not necessarily affect the relative terms of trade between the two countries in such a way as to account for the increase in per-capita Argentinean income relative to Jamaican observed in historical data.

namics of quality upgrading in consumption, in a context of worldwide growth and international trade, may cause world demand to continuously shift towards those economies that produce commodities offering larger scope for quality improvements. This result may hold, nonetheless, under more general assumptions regarding preferences and technology, some of them already discussed in previous sections.

Concerning preferences, the essential feature is that the marginal utility of quality upgrading rises with income fast enough relative to the marginal cost of quality upgrading. With respect to the technology itself, somewhat restrictive assumptions such as: (i) the monotonicity of $A(v)$ in Assumption 1 or (ii) the fact that functions $a(v)$ and $a^*(v)$ are both increasing in v, could be relaxed. A non-monotonic $A(v)$ will certainly lead to richer general equilibrium responses to worldwide technological growth. Yet, the key finding that the economy specialising in the goods with *lowest* cost of quality upgrading will *eventually* experience faster growth through the improvement of its terms of trade would still obtain. On the other hand, letting $a(v)$ and $a^*(v)$ behave in an unrestricted way may give rise to changes and switches in the order of quality upgrading across goods. However, again, the goods with relatively low elasticity of quality upgrading, i.e. those with relatively low $\eta(v)$, will *eventually* become those that experience relatively fast quality upgrading, just as it occurred in our benchmark model.

We conducted a number of empirical exercises which suggest that the central theoretical predictions of our model find some empirical support in the trade data. In particular, we find that exports, both at a world- and country-level of aggregation, correlate with world income changes in a way that is consistent with our theory. Our tests followed two stylised reduced-form approaches. The first approach shows that long-ladder goods tend to attract increasing world expenditure shares with rising world income. The second approach shows that exports from countries specialising in the production of long-ladder goods increase relatively more when world income rises. In addition, our finding that economies' initial specialisation correlates with future exports growth in a context of world income growth is suggestive of the fact that initial comparative advantage may lead to uneven future exports growth when goods differ in their scope for quality upgrading.

Finally, here, we have focused on illustrating how our theory may shed light on historical cases where comparative advantage emerged exogenously, for example as a result of geographical conditions. Other issues, including what determines the relative successes of economies with similar comparative advantage, and why richer countries tend to trade among themselves more than they do with poorer economies, are the subject of ongoing research.

Appendices

A First-Order Conditions for Consumption Choice in H

The optimisation problem in (5) yields the following first-order conditions (where μ represents the Lagrange multiplier associated to the budget constraint and $\{\lambda_v\}_{v\in\mathbb{V}}$ denote those associated to the constraints $\{q_v \geq 1\}_{v\in\mathbb{V}}$):

$$\ln\left(\frac{\beta_v w}{\kappa^{-1}\alpha(v)\, q_v^{\eta(v)}}\right) - \eta(v) + \lambda_v = 0,\ \forall v \in \mathbb{V}; \tag{19}$$

$$\frac{q_v}{\beta_v} - \mu = 0,\ \forall v \in \mathbb{V}; \tag{20}$$

$$q_v - 1 \geq 0,\ \lambda_v \geq 0,\ \text{and } \lambda_v (q_v - 1) = 0,\ \forall v \in \mathbb{V}; \tag{21}$$

$$1 - \int_{\mathbb{V}} \beta_v\, dv = 0. \tag{22}$$

From (20), it follows that $\beta_v = q_v/\mu$. Then, replacing this last expression into (22) leads to $\int_{\mathbb{V}} q_z\, dz = \mu$, from where the condition (6) immediately obtains by using again (20).

By using the condition (6), we can rewrite (19) as:

$$\lambda_v = \eta(v) + \ln[\alpha(v)/w] - \ln\kappa + \ln Q + [\eta(v) - 1]\ln q_v. \tag{23}$$

The expression in (23) will be used in many of the following proofs.

B Optimal Consumption Choice in F

Bearing in mind Assumption 1, we can write down the optimisation problem faced by a representative individual from F as follows:

$$\begin{aligned}
&\max_{\{x^*_{vq}\}_{(v,q)\in\mathbb{V}\times\mathbb{Q}}} && U^* = \int_{\mathbb{V}} q^*_v \ln\left(\frac{w^*\beta^*_v}{p_v}\right) dv;\\
&\text{subject to:} && \textstyle\int_{\mathbb{V}} \beta^*_v dv = 1,\\
& && q^*_v \geq 1,\ \forall v \in \mathbb{V},\\
& && p_{vq} = \kappa^{-1} q^{\eta(v)} \alpha(v),\quad \forall (v,q) \in \mathbb{V}\times\mathbb{Q}.
\end{aligned}$$

Lemma 4 holds for x^*_{vq} in a similar fashion as for x_{vq}. Hence, we can re-state the problem specified above in terms of q^*_v and β^*_v, as it was previously done for H (where q^*_v now denotes the quality of good v consumed, in the optimum, in F). This way, we can obtain the following first-order conditions, which constitute the analogous versions for F of (6) and (23), respectively:

$$\beta_v^* = \frac{q_v^*}{\int_{\mathbb{V}} q_z^* \, dz}, \; \forall v \in \mathbb{V}, \tag{24}$$

$$\lambda_v^* = \eta(v) + \ln[\alpha(v)/w^*] - \ln\kappa + \ln Q^* + [\eta(v) - 1]\ln q_v^*. \tag{25}$$

Given the first-order conditions in (24) and (25), all the ensuing results found in Section 3 follow through in qualitative terms. In particular, we can derive functions $\{q_v^*\}_{v\in\mathbb{V}}$ and $\{\beta_v^*\}_{v\in\mathbb{V}}$ displaying identical qualitative properties as their *counterparts* in H, that is $\{q_v\}_{v\in\mathbb{V}}$ and $\{\beta_v\}_{v\in\mathbb{V}}$, in terms of Lemmas 1 - 2 and Proposition 1. Furthermore, we can similarly find the threshold $\underline{\kappa}^*$ for the worldwide aggregate-productivity parameter, which splits F in the regimes of *subsistence-economy* and *modern-economy*; both exhibiting analogous properties as described for H.[28] Finally, likewise for H in Corollary 1, for F the following holds:

Corollary 2

Let $\vartheta^*(v) \equiv \int_0^v \beta_z^* \, dz$. *Then:*

(i) $\forall\kappa \in (0, \underline{\kappa}^*) \;\Rightarrow\; \partial\vartheta^*(v)/\partial\kappa = 0, \; \forall\, v \in \mathbb{V}$*;*

(ii) $\forall\kappa \geq \underline{\kappa}^* \;\Rightarrow\; \partial\vartheta^*(v)/\partial\kappa > 0, \; \forall v \in \mathbb{V}$; *with strict inequality if* $v < 1$.

C Omitted Proofs

Proof of Lemma 1.

Suppose $q_{\underline{v}} < q_{\overline{v}}$. Since by definition $q_{\underline{v}} \geq 1$, then $q_{\overline{v}} > 1$, hence (23) paired with (21) yield:

$$\eta(\underline{v}) + \ln[\alpha(\underline{v})/w] - \ln(\kappa/Q) \geq 0,$$

while:

$$\eta(\overline{v}) + \ln[\alpha(\overline{v})/w] - \ln(\kappa/Q) + [\eta(\overline{v}) - 1]\ln q_{\overline{v}} = 0.$$

Thus:

$$\eta(\underline{v}) + \ln\alpha(\underline{v}) \geq \eta(\overline{v}) + \ln\alpha(\overline{v}) + [\eta(\overline{v}) - 1]\ln q_{\overline{v}}.$$

This last equality in turn leads to:

$$[\eta(\overline{v}) - \eta(\underline{v})] + \ln[\alpha(\overline{v})/\alpha(\underline{v})] + [\eta(\overline{v}) - 1]\ln q_{\overline{v}} \leq 0,$$

[28] From Section 4, it is straightforward to observe that, given Assumption 1, $\underline{\kappa}^* = \underline{\kappa}$.

which cannot possibly hold if $q_{\overline{v}} > 1$, as its left-hand side would then be strictly positive. Therefore, it must necessarily be the case that $q_{\underline{v}} \geq q_{\overline{v}}$.

Suppose now $q_{\underline{v}} = q_{\overline{v}} > 1$. In this case, (23) in conjunction with (21) yield:

$$\eta\left(\underline{v}\right) + \ln\alpha\left(\underline{v}\right) + \left[\eta\left(\underline{v}\right) - 1\right]\ln q_{\overline{v}} = \eta\left(\overline{v}\right) + \ln\alpha\left(\overline{v}\right) + \left[\eta\left(\overline{v}\right) - 1\right]\ln q_{\overline{v}} = 0.$$

This last equality in turn leads to:

$$-\left[\eta\left(\overline{v}\right) - \eta\left(\underline{v}\right)\right]\left(1 + \ln q_{\overline{v}}\right) = \ln\left[\alpha\left(\overline{v}\right)/\alpha\left(\underline{v}\right)\right].$$

However, this last equality cannot possibly hold since its right-hand side is strictly positive, while the left-hand side is negative. As a result, $q_{\underline{v}} > q_{\overline{v}}$ must necessarily hold when $q_{\underline{v}} > 1$. ■

Proof of Lemma 2.

Part (i). Proof follows immediately from noting that Lemma **??** implies that, whenever $\kappa \in (0, \underline{\kappa})$, $q_v = 1$ must hold for all $v \in \mathbb{V}$. Thus, whenever $\kappa \in (0, \underline{\kappa})$, $\partial q_v/\partial\kappa = 0$ for all $v \in \mathbb{V}$.

Part (ii). Firstly, notice that, since $q_v = 1$ must hold for all $v \notin \mathbb{L}$, a proof analogous to that of Part (i) of this Lemma applies for all goods in this subset. Secondly, for any $v \in \mathbb{L}$, differentiating (31) with respect to κ yields:

$$\frac{dq_v}{d\kappa} = \frac{\eta\left(0\right) - 1}{\eta\left(v\right) - 1}\left[\frac{e^{\eta(0)}\alpha\left(0\right)}{e^{\eta(v)}\alpha\left(v\right)}\right]^{\frac{1}{\eta(v)-1}}\left(q_0\right)^{\frac{\eta(0)-\eta(v)}{\eta(v)-1}}\frac{dq_0}{d\kappa},\ \forall v \in \mathbb{L}.$$

Using again (31), the equation above can be written:

$$\frac{dq_v}{d\kappa} = \frac{\eta\left(0\right) - 1}{\eta\left(v\right) - 1}\frac{q_v}{q_0}\frac{dq_0}{d\kappa},\ \forall v \in \mathbb{L}. \tag{26}$$

(Since $\eta\left(\cdot\right) > 1$, notice that $dq_v/d\kappa$ and $dq_0/d\kappa$ must then share the same sign, for all $v \in \mathbb{L}$). Given that:

$$Q = 1 - \tilde{v}(\kappa) + \int_0^{\tilde{v}(\kappa)} q_z\, dz,$$

it follows that:

$$\frac{dQ}{d\kappa} = \int_0^{\tilde{v}(\kappa)}\frac{dq_z}{d\kappa}dz = \frac{1}{q_0}\left(\int_0^{\tilde{v}(\kappa)}\frac{\eta\left(0\right) - 1}{\eta\left(z\right) - 1}q_z dz\right)\frac{dq_0}{d\kappa}.$$

Applying (23) to $v = 0$ when $\lambda_0 = 0$ yields: $q_0 = \left[a\left(0\right)e^{\eta(0)}Q\right]^{-\frac{1}{\eta(0)-1}}\kappa^{\frac{1}{\eta(0)-1}}$. Thus:

$$\frac{dq_0}{d\kappa} = \frac{q_0}{\eta\left(0\right) - 1}\frac{Q}{\kappa}\left(1 - \tilde{v} + \int_0^{\tilde{v}(\kappa)}\frac{\eta\left(z\right)}{\eta\left(z\right) - 1}q_z\, dz\right)^{-1} > 0.$$

Therefore, from (26) it follows that $dq_v/d\kappa > 0$, $\forall v \in \mathbb{L}$ must also hold. Finally, from (26) it immediately follows that $dq_{\underline{v}}/d\kappa = dq_{\overline{v}}/d\kappa = 0$ if $\underline{v}, \overline{v} \notin \mathbb{L}$, and $dq_{\underline{v}}/d\kappa > dq_{\overline{v}}/d\kappa = 0$ if $\underline{v} \in \mathbb{L}$ and $\overline{v} \notin \mathbb{L}$. For $\underline{v}, \overline{v} \in \mathbb{L}$, such that $\underline{v} < \overline{v}$, (26) leads to:

$$\frac{dq_{\underline{v}}}{d\kappa} = \frac{\eta(0) - 1}{\eta(\underline{v}) - 1} \frac{q_{\underline{v}}}{q_0} \frac{dq_0}{d\kappa} > \frac{\eta(0) - 1}{\eta(\overline{v}) - 1} \frac{q_{\overline{v}}}{q_0} \frac{dq_0}{d\kappa} = \frac{dq_{\overline{v}}}{d\kappa},$$

since by assumption $\eta(\underline{v}) < \eta(\overline{v})$ and, from Lemma 1, $q_{\underline{v}} > q_{\overline{v}}$. ■

Proof of Proposition 1.

Part (i). Proof follows immediately from noting that Lemma 2 implies that, whenever $\kappa \in (0, \underline{\kappa})$, $\partial q_v/\partial\kappa = 0$ must hold for all $v \in \mathbb{V}$. Thus, whenever $\kappa \in (0, \underline{\kappa})$, $\partial\beta_v/\partial\kappa = 0$ for all $v \in \mathbb{V}$.

Part (ii). Firstly, suppose that that $\underline{v} \notin \mathbb{L}$. Then, from Lemma 1 it must also be that $\overline{v} \notin \mathbb{L}$. Hence from Lemma 2 $dq_{\underline{v}}/d\kappa = dq_{\overline{v}}/d\kappa = 0$, implying in turn that $d\beta_{\underline{v}}/d\kappa = d\beta_{\overline{v}}/d\kappa$. Secondly, suppose that $\underline{v} \in \mathbb{L}$. Considering the definition of average quality, taking logarithms and differentiating (6) with respect to κ yields:

$$(d\beta_v/d\kappa)/\beta_v = (dq_v/d\kappa)/q_v - (dQ/d\kappa)/Q.$$

Using (26), we can write:

$$\frac{dq_{\underline{v}}}{d\kappa} \frac{1}{q_{\underline{v}}} = \frac{\eta(0) - 1}{\eta(\underline{v}) - 1} \frac{dq_0}{d\kappa} \frac{1}{q_0} > \frac{\eta(0) - 1}{\eta(\overline{v}) - 1} \frac{dq_0}{d\kappa} \frac{1}{q_0} \geq \frac{dq_{\overline{v}}}{d\kappa} \frac{1}{q_{\overline{v}}}; \tag{27}$$

where the last (weak) inequality stems from the fact that if $\overline{v} \in \mathbb{L}$ then (26) holds for $\overline{v}$, whereas if $\overline{v} \notin \mathbb{L}$ then $dq_{\overline{v}}/d\kappa = 0$. It follows then that:

$$\frac{d\beta_{\underline{v}}}{d\kappa} \frac{1}{\beta_{\underline{v}}} > \frac{d\beta_{\overline{v}}}{d\kappa} \frac{1}{\beta_{\overline{v}}}. \tag{28}$$

Finally, using (28), the claim trivially follows by noting that, from Lemma 1 in conjunction with (6), $\beta_{\underline{v}} > \beta_{\overline{v}}$ must always hold. ■

Proof of Corollary 1.

Preliminarily, recall $\int_{z \in \mathbb{V}} \beta_z \, dz = 1$, which implies $\int_0^1 (\partial\beta_z/\partial\kappa) \, dz = 0$.[29]

Part (i). Claim immediately follows since, whenever $\kappa < \underline{\kappa}$, $\partial\beta_z/\partial\kappa = 0$ for all $z \in \mathbb{V}$.

Part (ii). Note first that when $\kappa \geq \underline{\kappa}$, the set $\mathbb{J} \neq \emptyset$. As a result, from Proposition 1, Part (i), it follows that $\int_0^v (\partial\beta_z/\partial\kappa) \, dz > \int_v^1 (\partial\beta_z/\partial\kappa) \, dz$. Then, since $\int_0^v (\partial\beta_z/\partial\kappa) \, dz + \int_v^1 (\partial\beta_z/\partial\kappa) \, dz = 0$, we must necessarily have that $\int_0^v (\partial\beta_z/\partial\kappa) \, dz > 0$. ■

[29] Note that it is then trivial to observe that $\partial\vartheta(1)/\partial\kappa = 0$, $\forall\kappa > 0$.

D Auxiliary Derivations and Proofs

Lemma 3

Let $\underline{\kappa} \equiv a(0)\exp[\eta(0)]$. *Then:*

(i) $\forall \kappa \in (0, \underline{\kappa}) \ \Rightarrow \ \mathbb{L} = \emptyset$;

(ii) $\forall \kappa \geq \underline{\kappa} \ \Rightarrow \ \mathbb{L} = [0, \tilde{v}(\kappa)]$; *where* $\tilde{v}(\kappa) : [\underline{\kappa}, \infty) \to [0,1]$; $\tilde{v}(\underline{\kappa}) = 0$; $\tilde{v}'(\kappa) \geq 0$, *with strict inequality if* $\tilde{v}(\kappa) < 1$.

Proof.

Part (i). When $\kappa \in (0, \underline{\kappa})$, conditions stipulated in (21) and (23) applied on $v = 0$ entail that: $q_0 = 1$ and $\lambda_0 > 0$. As a result, from Lemma 1 it follows that $q_v = 1$, $\forall v \in \mathbb{V}$. Therefore, since $a'(v) \geq 0$ and $\eta'(v) > 0$, again from (23), $\lambda_v > 0$ for all $v \in \mathbb{V}$ obtains, and thus $\mathbb{L} = \emptyset$.

Part (ii). Firstly, note that (23) applied on $v = 0$, in conjunction Lemma 1, implies that when $\kappa = \underline{\kappa}$, then $\lambda_0 = 0$ and $q_0 = 1$. Then, Lemma 1 implies $Q = 1$. Using these results in (23) yields:

$$\lambda_v = \eta(v) + \ln[\alpha(v)/w] - \ln \underline{\kappa},$$

implying that $\lambda_v > 0$ for all $v \in (0,1]$. As a result, the set $\mathbb{L} = \emptyset$, meaning that $\tilde{v}(\underline{\kappa}) = 0$. Secondly, notice that, from Lemma 4 below, $\partial\Phi_{0,v}(v)/\partial v < 0$ and $\partial\Upsilon_{0,v}(v)/\partial v < 0$ since $a'(v) \geq 0$ and $\eta'(v) > 0$, hence the set $\mathbb{L} \subseteq \mathbb{V}$ comprises the lower-indexed goods in $\mathbb{V}$, with $\tilde{v}(\underline{\kappa})$ representing its upper bound. Given Lemma 1 and Lemma 5 below, the aggregate quality index can be written as follows:

$$Q = 1 - \tilde{v}(\kappa) + \int_0^{\tilde{v}(\kappa)} q_v \, dv.$$

Furthermore, observe that, whenever $\tilde{v}(\kappa) < 1$:

$$\ln(\kappa/Q) = \eta(\tilde{v}(\kappa)) + \ln[\alpha(\tilde{v}(\kappa))/w]$$

must hold in equilibrium. This last condition yields, after some simple algebra:

$$Q = \kappa w \exp[-\eta(\tilde{v})]/\alpha(\tilde{v}).$$

In addition to that, because of Lemma 1, in equilibrium:

$$[\eta(v) - 1]\ln q_v = \ln(\kappa/Q) - \eta(v) - \ln[\alpha(v)/w]$$

must hold for any $v \leq \tilde{v}(\kappa)$. By using the former in the latter, after some algebra, we may obtain:

$$q_v = q_v(\tilde{v}(\kappa)) \equiv \left[\frac{\alpha(\tilde{v}(\kappa))}{\alpha(v)}\right]^{\frac{1}{\eta(v)-1}} \exp\left[\frac{\eta(\tilde{v}(\kappa)) - \eta(v)}{\eta(v)-1}\right], \ \forall v \in [0, \tilde{v}(\kappa)]. \tag{29}$$

In equilibrium, it must be the case that:

$$\kappa w \exp[-\eta(\tilde{v}(\kappa))] / a(\tilde{v}(\kappa)) = 1 - \tilde{v}(\kappa) + \int_0^{\tilde{v}(\kappa)} q_v(\tilde{v}(\kappa))\, dv, \tag{30}$$

where the right hand-side of (30) uses (29). Computing the total differentiation of (30), yields after some algebra:[30]

$$\frac{Q}{\kappa} d\kappa = \left[\frac{\alpha'(\tilde{v}(\kappa))}{\alpha(\tilde{v}(\kappa))} + \eta'(\tilde{v}(\kappa))\right]\left[Q + \int_0^{\tilde{v}(\kappa)} \frac{q_v}{\eta(v)-1} dv\right] d\tilde{v},$$

leading finally to:

$$\frac{d\tilde{v}}{d\kappa} = \left[\frac{\kappa}{Q}\left(\frac{\alpha'(\tilde{v}(\kappa))}{\alpha(\tilde{v}(\kappa))} + \eta'(\tilde{v}(\kappa))\right)\left(1 - \tilde{v}(\kappa) + \int_0^{\tilde{v}(\kappa)} \frac{\eta(v)}{\eta(v)-1} q_v dv\right)\right]^{-1} > 0.$$

where the last inequality follows from the properties of the functions $\alpha(\cdot)$ and $\eta(\cdot)$. ■

Lemma 4

The optimal quality q_v of any good $v \in \mathbb{V}$ can be written as follows:

$$q_v = \max\left\{\Phi_{0,v}(q_0)^{\Upsilon_{0,v}}, 1\right\}; \tag{31}$$

where:

$$\Phi_{0,v} \equiv \left[\frac{e^{\eta(0)}\alpha(0)}{e^{\eta(v)}\alpha(v)}\right]^{\frac{1}{\eta(v)-1}} > 0, \ \textit{and } \Upsilon_{0,v} \equiv \frac{\eta(0)-1}{\eta(v)-1} > 0.$$

Proof.

Recall that $q_v = 1$, $\forall v \notin \mathbb{L}$. For all other goods, (23) in conjunction with (21) yield:

$$\eta(v) + \ln\alpha(v) + [\eta(v) - 1]\ln q_v = \eta(0) + \ln\alpha(0) + [\eta(0) - 1]\ln q_0, \ \forall v \in \mathbb{L}.$$

[30] For the rest of the proof, we will assume that the envelope function $\alpha(v)$ is differentiable at all points. It must be straightforward to observe, though, that the function $\alpha(v)$ is strictly increasing in v, since both $a(v)$ and $a^*(v)$ are strictly increasing in v, and that this monotonicity is sufficient to ensure monotonicity of $\widetilde{v}(\kappa)$, which is the important feature of $\widetilde{v}(\kappa)$ that we require in our model.

Isolating $[\eta(v)-1]\ln q_v$, and applying exponentials to both sides gives:

$$(q_v)^{\eta(v)-1} = \frac{e^{\eta(0)}}{e^{\eta(v)}}\frac{\alpha(0)}{\alpha(v)}(q_0)^{\eta(0)-1},\ \forall v \in \mathbb{L}.$$

Finally, raising both sides to the power $[\eta(v)-1]^{-1}$, and considering Lemma 1, (31) obtains. ■

Lemma 5

If $\tilde{v}(\kappa) < 1$*, then* $q_{\tilde{v}(\kappa)} = 1$*.*

Proof.

By definition of $\mathbb{L}$, $\lambda_{\tilde{v}(\kappa)} = 0$. Thus, the condition (23) applied on $\tilde{v}(\kappa)$ yields:

$$\eta(\tilde{v}(\kappa)) + \ln[\alpha(\tilde{v}(\kappa))/w] - \ln\kappa + \ln Q = -[\eta(\tilde{v}(\kappa)) - 1]\ln q_{\tilde{v}(\kappa)} \tag{32}$$

Suppose now that $q_{\tilde{v}(\kappa)} > 1$, and take some $\varepsilon \in (0, 1-\tilde{v}(\kappa)]$. Then, since $v = \tilde{v}(\kappa) + \varepsilon \notin \mathbb{L}$, it must be the case that:

$$\eta(\tilde{v}(\kappa)+\varepsilon) + \ln[\alpha(\tilde{v}(\kappa)+\varepsilon)/w] - \ln\kappa + \ln Q = \lambda_{\tilde{v}(\kappa)+\varepsilon}. \tag{33}$$

Then, by continuity of $\eta(\cdot)$ and $\alpha(\cdot)$, and using the result in (32), we must have:

$$\lim_{\varepsilon\to 0}\{\eta(\tilde{v}(\kappa)+\varepsilon) + \ln[\alpha(\tilde{v}(\kappa)+\varepsilon)/w] - \ln\kappa + \ln Q\} = -[\eta(\tilde{v}(\kappa)) - 1]\ln q_{\tilde{v}(\kappa)} < 0.$$

Hence, $q_{\tilde{v}(\kappa)} > 1$ cannot possibly hold when $\tilde{v}(\kappa) < 1$ as it would imply that $\lambda_{\tilde{v}(\kappa)+\varepsilon} < 0$ in (33) for $\varepsilon \to 0$, violating (21). ■

Proof of $\partial\vartheta(m)/\partial w \leq 0$**.**

Suppose first that $\tilde{v} < m$. Then, $\mathbb{L} \subset [0, m)$. Differentiating (23) with respect to w yields:

$$\frac{\eta(v)-1}{q_v}\frac{\partial q_v}{\partial w} + \frac{1}{Q}\frac{\partial Q}{\partial w} = 0,\ \forall v \in \mathbb{L}. \tag{34}$$

Furthermore, from (31) it follows that:

$$\frac{\partial q_v}{\partial w} = \frac{\eta(0)-1}{\eta(v)-1}\frac{q_v}{q_0}\frac{\partial q_0}{\partial w},\ \forall v \in \mathbb{L}. \tag{35}$$

Since $\partial Q/\partial w = \int_0^{\tilde{v}}(\partial q_z/\partial w)\,dz$, combining (34) and (35) yields:

$$\left(1 - \tilde{v} + \int_0^{\tilde{v}}\frac{\eta(z)}{\eta(z)-1}q_z dz\right)\frac{\eta(0)-1}{q_0}\frac{1}{Q}\frac{\partial q_0}{\partial w} = 0 \quad\Rightarrow\quad \frac{\partial q_0}{\partial w} = 0,$$

Therefore, using again (35), $\partial q_v/\partial w = 0$ for all $v \in [0,\tilde{v}]$ obtains. In addition, because of Lemma 1, it must thus be the case that $\partial q_v/\partial w = 0$ holds as well for all $v \in (\tilde{v},1]$. Finally, recalling (6) it then follows that $\partial \beta_v/\partial w = 0$ for all $v \in \mathbb{V}$, which in turn implies that $\partial \vartheta(m)/\partial w = 0$.

Suppose now that $\tilde{v} \geq m$. Differentiating (23) with respect to w now yields:

$$\frac{\eta(v)-1}{q_v}\frac{\partial q_v}{\partial w} + \frac{1}{Q}\frac{\partial Q}{\partial w} = \begin{cases} 0, & \forall v \in [0,m) \\ 1/w, & \forall v \in [m,\tilde{v}] \end{cases} \tag{36}$$

From (36) it follows that a necessary condition for $\partial \vartheta(m)/\partial w > 0$ to hold is that $\partial Q/\partial w < 0$.[31] However, (36) means that if $\partial Q/\partial w < 0$, then $\partial q_v/\partial w > 0$ should hold for all $v \in [m,\tilde{v}]$. If $\tilde{v} = 1$, it must be straightforward to observe that $\partial Q/\partial w < 0$ cannot thus hold. Alternatively, if $\tilde{v} < 1$, then $\partial Q/\partial w < 0$ would require that $\partial q_v/\partial w < 0$ prevails for some $v \in (\tilde{v},1]$ which is not feasible either since it would lead to violating the constraint $q_v \leq 1$. As a result, $\partial Q/\partial w \geq 0$ must hold, which in turn implies $\partial \vartheta(m)/\partial w \leq 0$. ■

Proof of $\partial \vartheta^*(m)/\partial w < 0$.

Suppose first that $\tilde{v}^* < m$. Then, $\mathbb{L}^* \subset [0,m)$. Differentiating (23) – adjusted for representing an individual from F – with respect to w yields:

$$\frac{\eta(v)-1}{q_v^*}\frac{\partial q_v^*}{\partial w} + \frac{1}{Q^*}\frac{\partial Q^*}{\partial w} = -\frac{1}{w},\ \forall v \in \mathbb{L}^*. \tag{37}$$

In addition, from (31) it follows that:

$$\frac{\partial q_v^*}{\partial w} = \frac{\eta(0)-1}{\eta(v)-1}\frac{q_v^*}{q_0^*}\frac{\partial q_0^*}{\partial w},\ \forall v \in \mathbb{L}^*. \tag{38}$$

Combining (37) and (38) leads to:

$$\left(1-\tilde{v}^* + \int_0^{\tilde{v}^*}\frac{\eta(z)}{\eta(z)-1}q_z dz\right)\frac{\eta(0)-1}{q_0^*}\frac{1}{Q^*}\frac{\partial q_0^*}{\partial w} = -\frac{1}{w} \quad \Rightarrow \quad \frac{\partial q_0^*}{\partial w} < 0.$$

Hence, using again (38), $\partial q_v^*/\partial w < 0$ for all $v \in [0,\tilde{v}^*]$ obtains, which in turn implies $\partial Q^*/\partial w < 0$. Next, since for all $v \geq \tilde{v}^*$ the constraint $q_v^* \geq 1$ is binding, it must be the case that $\partial q_v^*/\partial w \geq 0$, $\forall v \in (\tilde{v}^*,1]$. As a result, because of (6), $\partial \beta_v^*/\partial w > 0$ for all $v \in [m,1]$ follows, which in turn implies $\partial \vartheta^*(m)/\partial w < 0$.

Suppose now $\tilde{v}^* \geq m$. Differentiating (23) with respect to w now yields:

$$\frac{\eta(v)-1}{q_v^*}\frac{\partial q_v^*}{\partial w} + \frac{1}{Q^*}\frac{\partial Q^*}{\partial w} = \begin{cases} -1/w, & \forall v \in [0,m) \\ 0, & \forall v \in [m,\tilde{v}^*] \end{cases} \tag{39}$$

[31] Otherwise, if $\partial Q/\partial w \geq 0$, (36) would imply that $\partial q_v/\partial w \leq 0$ for all $v \in [0,m)$. Recalling (6), it is then straightforward to observe that $\partial Q/\partial w \geq 0$ would mean $\partial \beta_v/\partial w \leq 0$ for all $v \in [0,m)$, which in turn leads to $\partial \vartheta(m)/\partial m \leq 0$.

Suppose $\partial Q^*/\partial w \geq 0$. From (39) it follows that $\partial q_v^*/\partial w < 0$ for all $v \in [0, \tilde{v}^*)$. Furthermore, Lemma 1 then implies that $\partial q_v^*/\partial w \leq 0$ for all $v \in [\tilde{v}^*, 1]$; as a result, $\partial Q^*/\partial w < 0$ must necessarily hold. Now, notice that if $\partial Q^*/\partial w < 0$, then (39) implies $\partial q_v^*/\partial w > 0$ for all $v \in [m, \tilde{v}^*]$. Moreover, in case $\tilde{v}^* < 1$, since $\forall v \in (\tilde{v}^*, 1]$ the constraint $q_v^* \geq 1$ is binding, $\partial q_v^*/\partial w \geq 0$ must necessarily hold for all $v \in (\tilde{v}^*, 1]$. As a result, if $\partial Q^*/\partial w < 0$, then $\partial \beta_v^*/\partial w > 0$ for all $v \in [m, 1]$, which in turn leads to $\partial \vartheta^* (m) / \partial w < 0$. ■

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